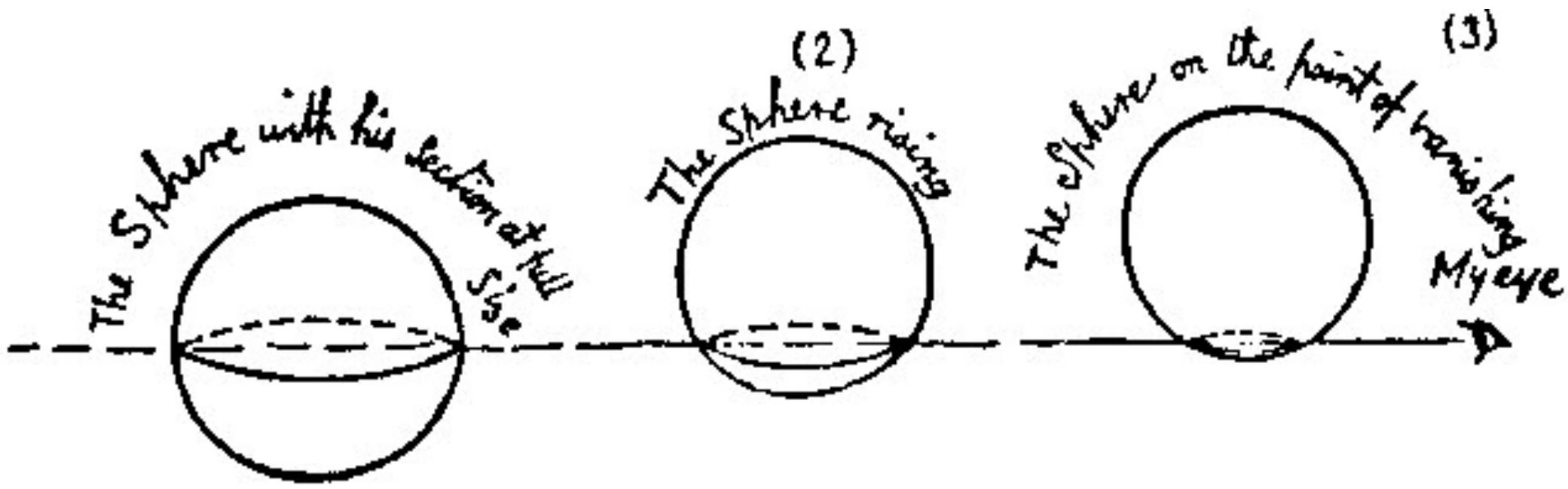
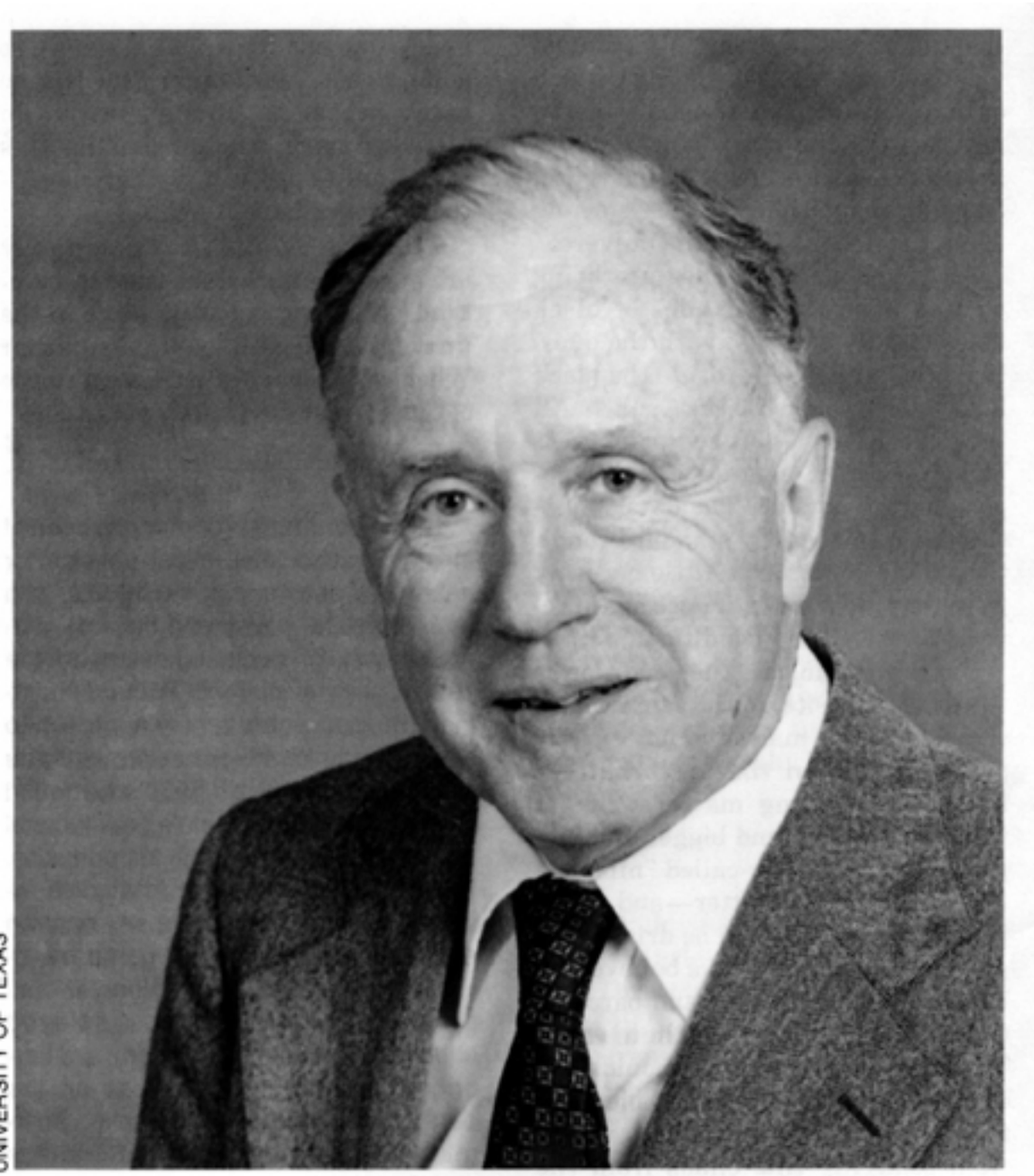
Time Dispersion in Quantum Mechanics

John Ashmead



International Association for Relativistic Dynamics - 2018 Yucatan

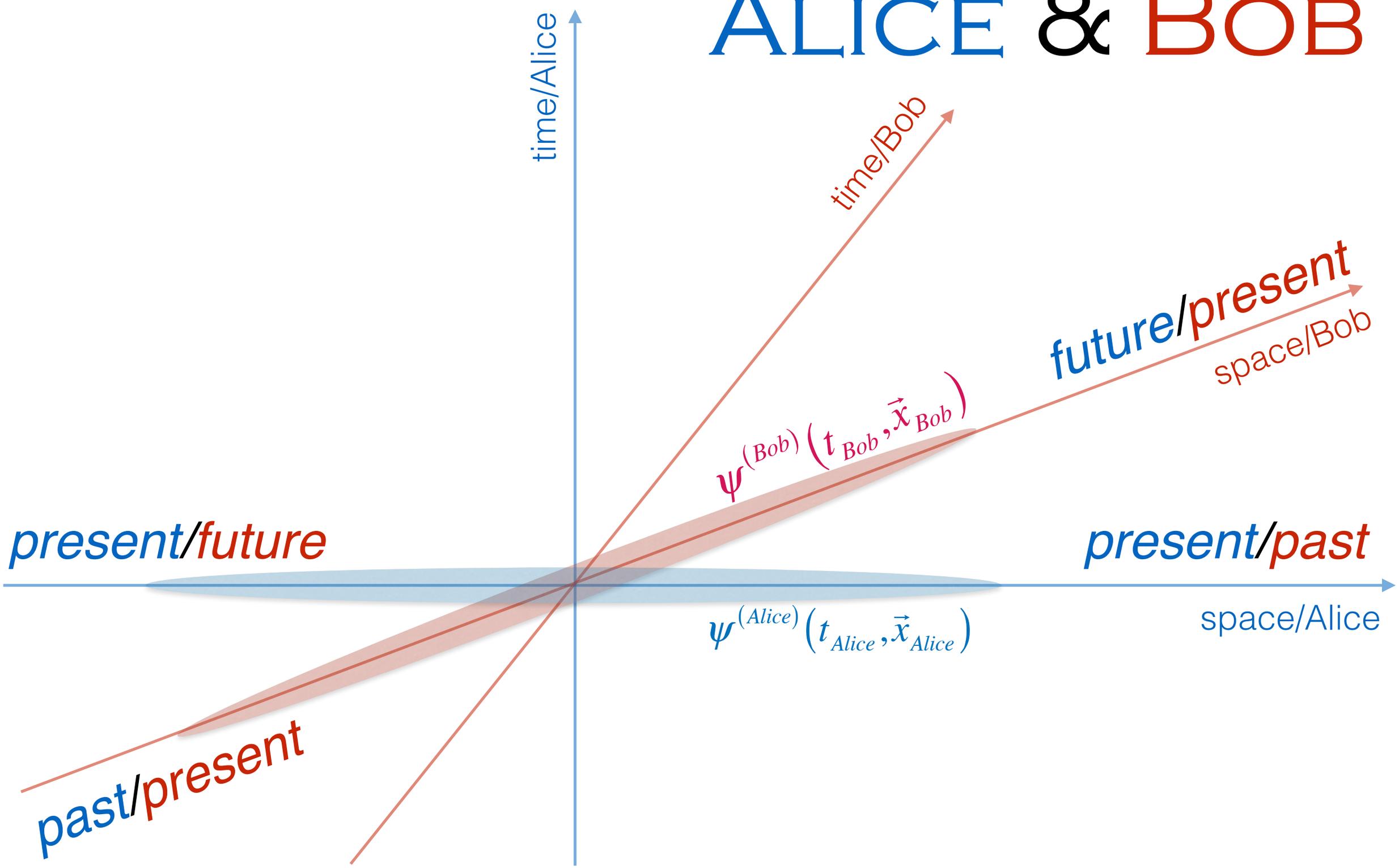
Radical Conservatism



Wheeler

"Wheeler's often unconventional vision of nature was grounded in reality through the principle of radical conservatism, which he acquired from Niels Bohr: Be *conservative* by sticking to well-established physical principles, but probe them by exposing their most *radical* conclusions." – Kip Thorne

Wave Functions for Alice & Bob

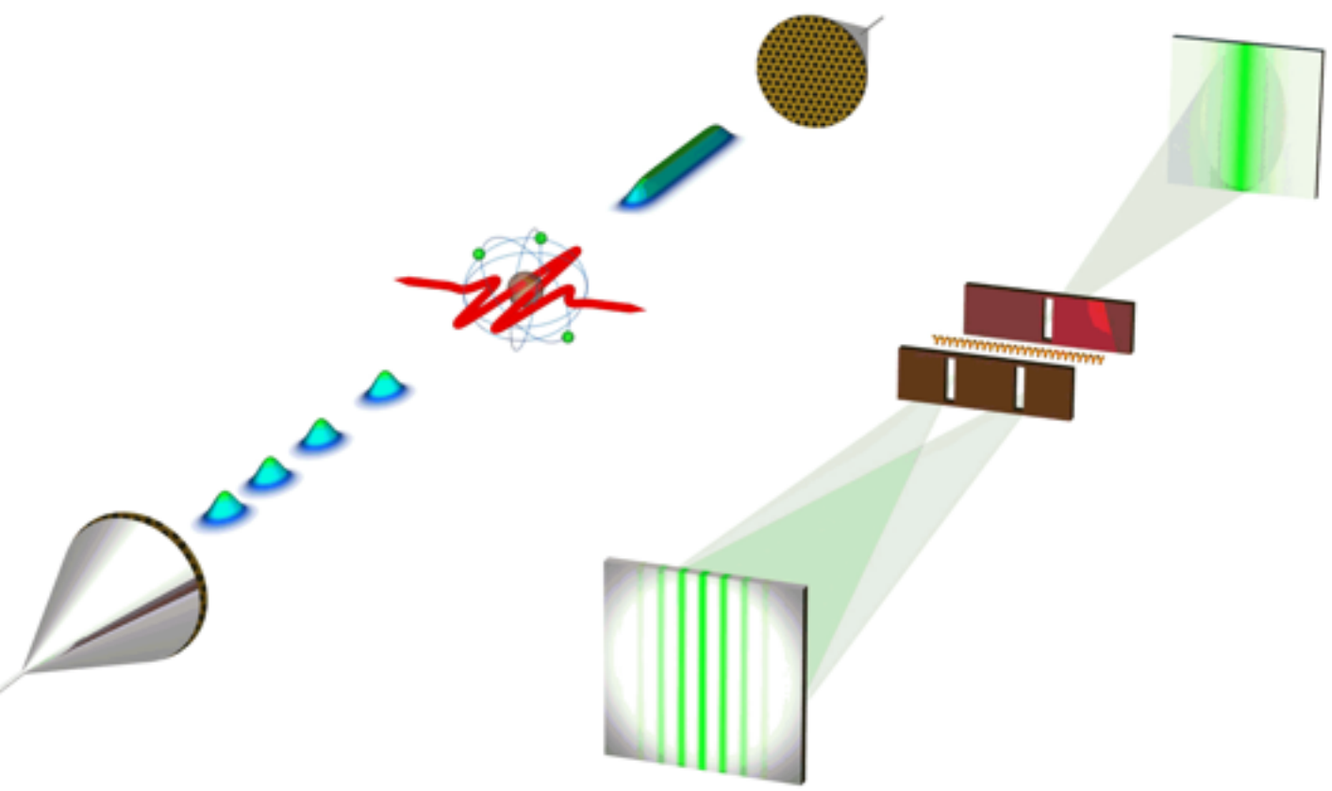


.177 ATTOSECONDS

$$a_0 = 5.310^{-11} m$$

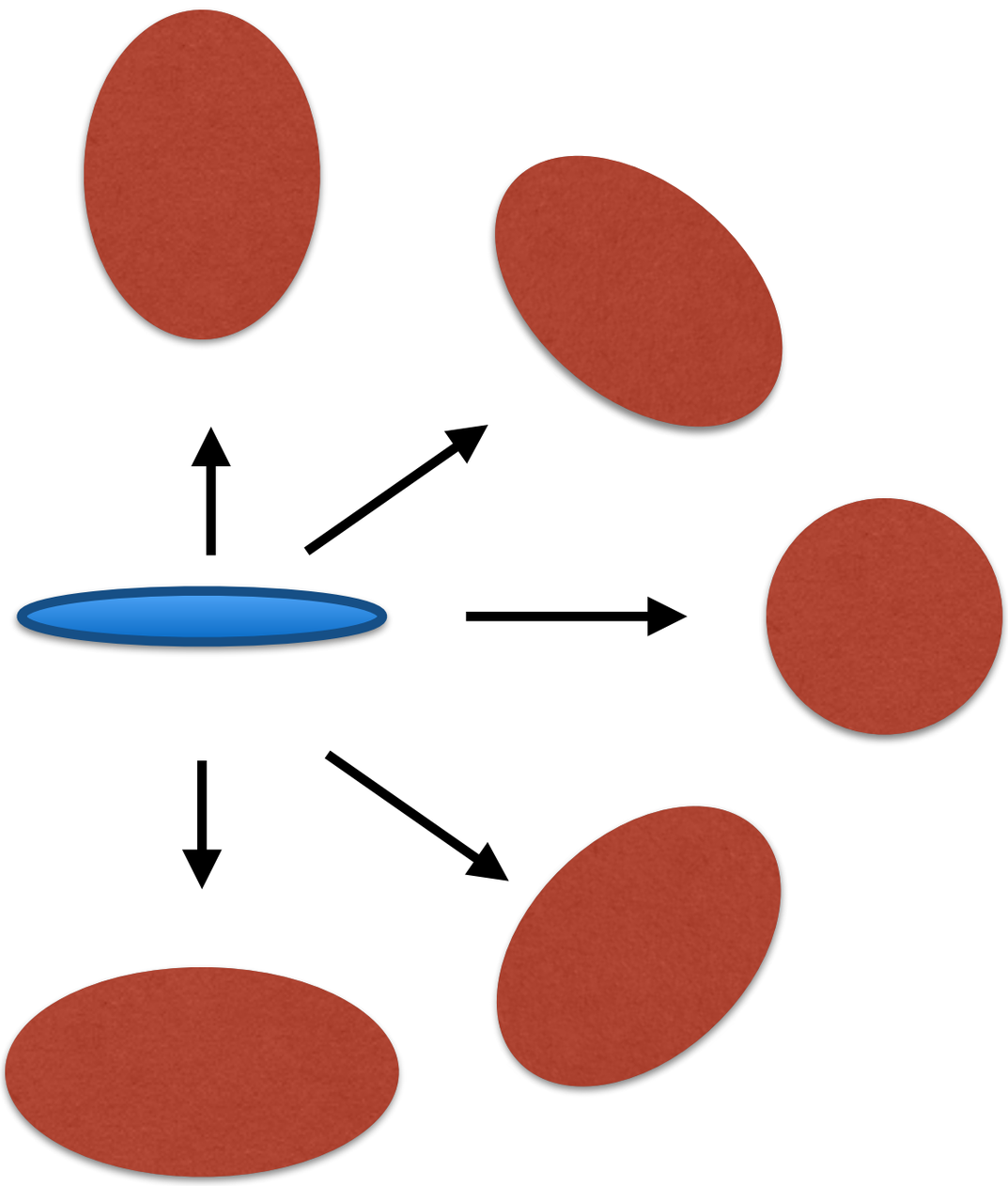
$$c = 310^8 \, m/s$$

$$a_{time} \equiv a_0 / c = 1.7710^{-19} = .177 as$$



- 500 as: Attosecond double-slit experiment - Lindner - 2005
- <1 as: Attosecond correlation dynamics - Ossiander - 2016

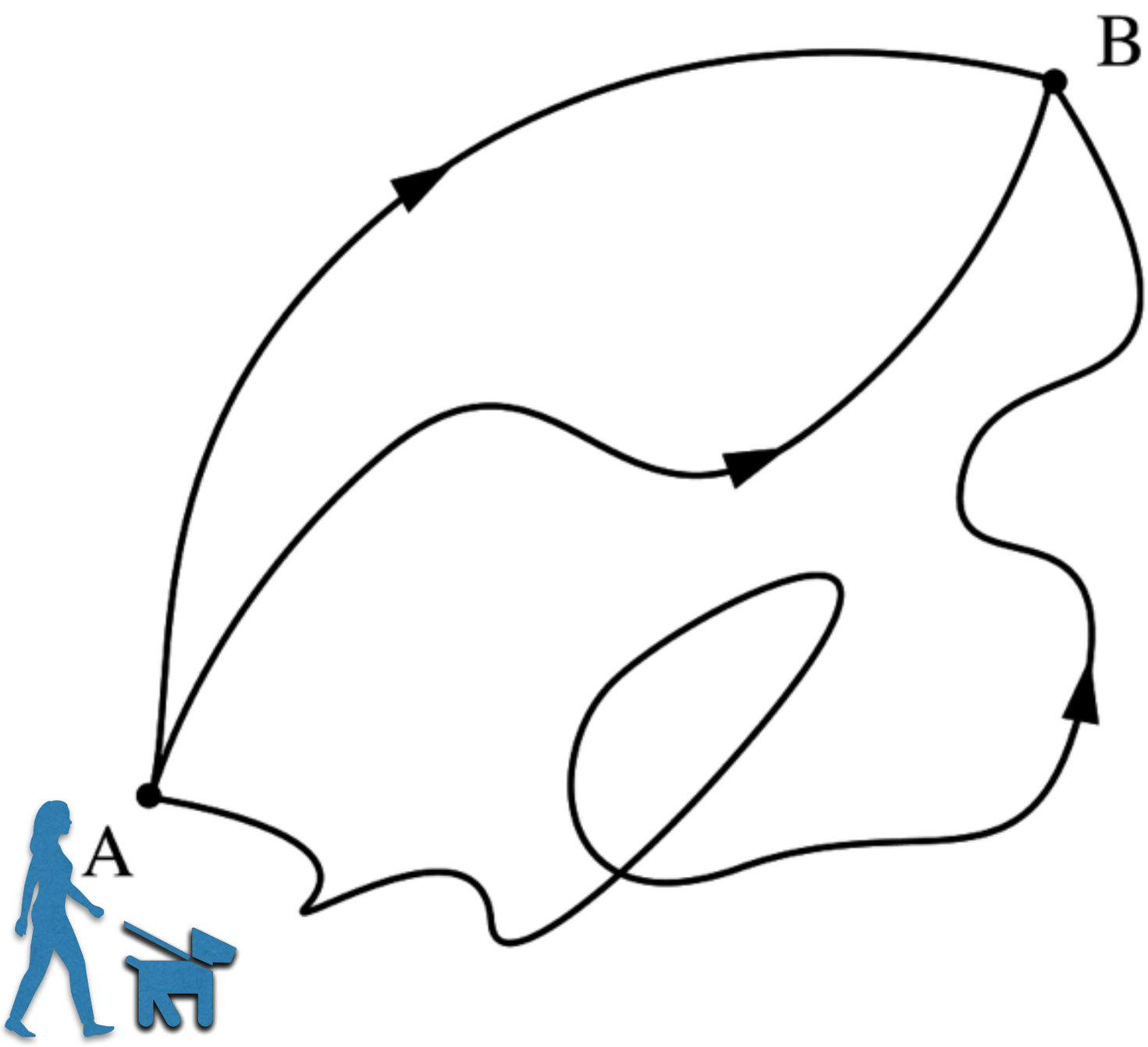
How to prove there is no dispersion in time?



- Work from symmetry & dimensional analysis
- Keep arguments simple
- Look for order of magnitude effects

What are path integrals?

$$\psi_B(x_B) = \int \mathcal{D}x \exp\left(\imath \int_A^B ds \mathcal{L}[x,\dot{x}]\right)\psi_A(x_A)$$

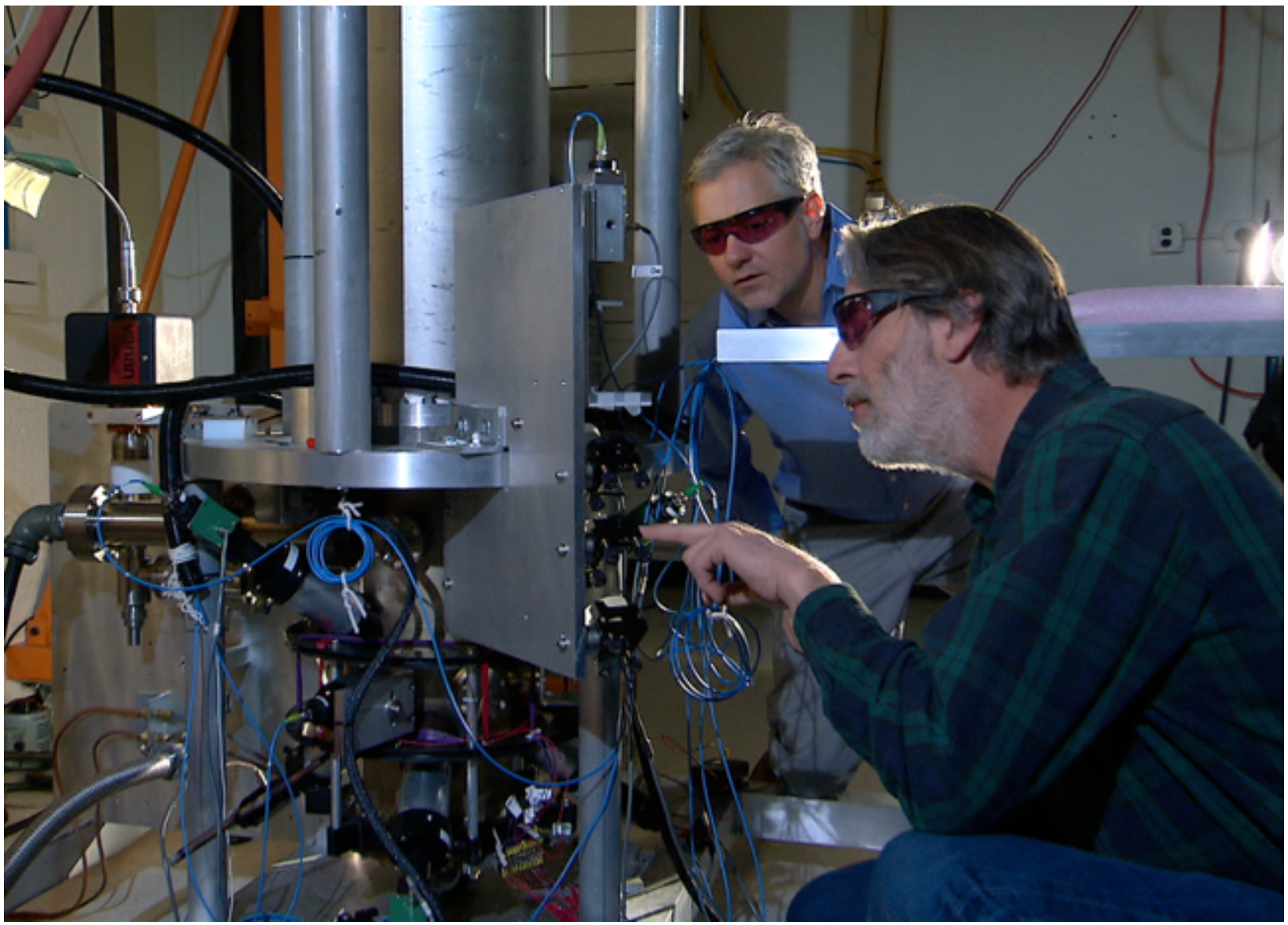


$$\mathcal{L}[x,\dot{x}] = T(x,\dot{x}) - V(x,\dot{x})$$

$$\mathcal{D}x = \prod_{n=1}^{N-1} dx_n$$

Laboratory time τ

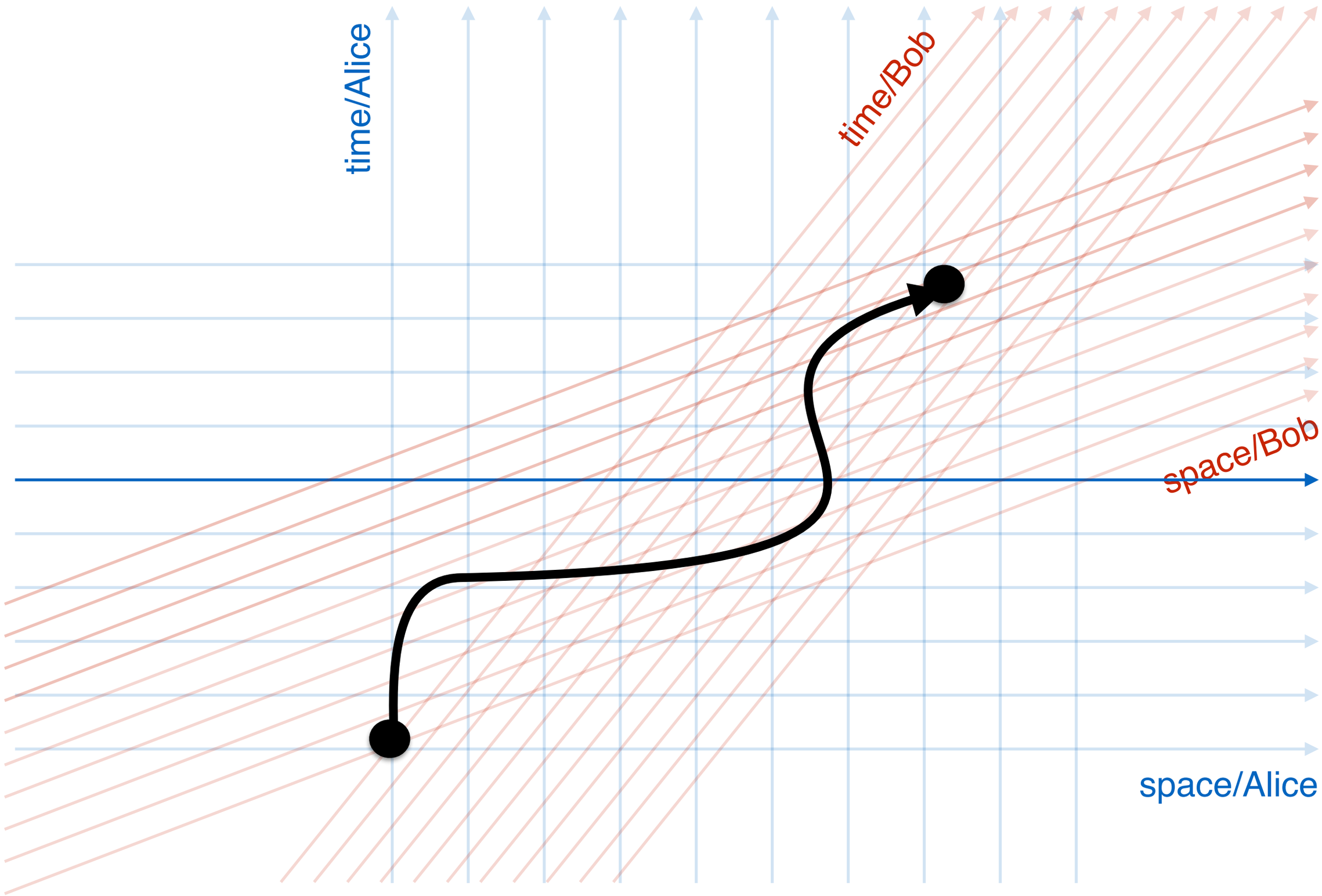
$$\int_A^B ds$$



$$\int_A^B ds$$

$$\int_A^B d\tau_{lab}$$

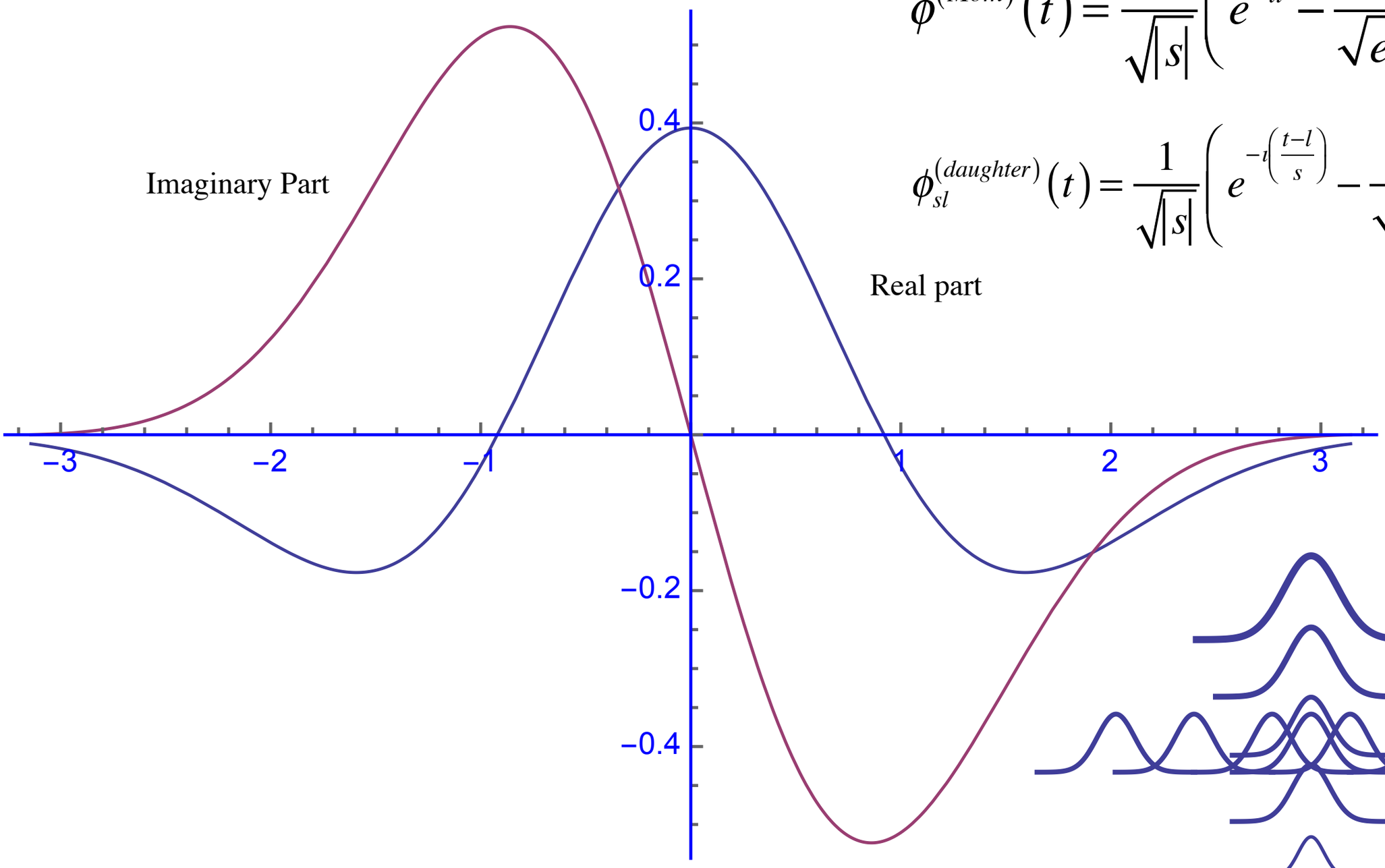
Coordinate time t

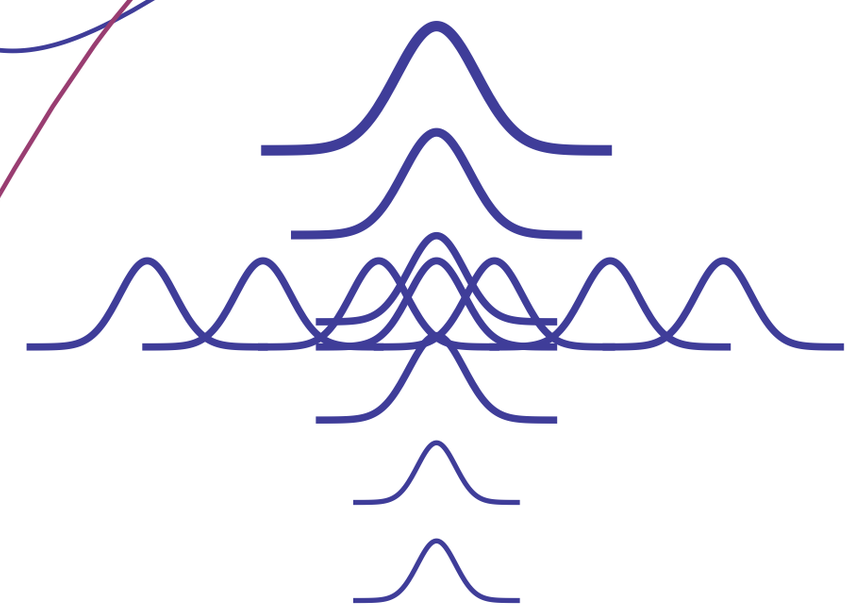


Morlet Wavelets

$$\phi^{(Mom)}(t) = \frac{1}{\sqrt{|s|}}\left(e^{-\imath t} - \frac{1}{\sqrt{e}}\right)e^{-\frac{1}{2}t^2}$$

$$\phi_{sl}^{(daughter)}(t) = \frac{1}{\sqrt{|s|}}\left(e^{-\imath\left(\frac{t-l}{s}\right)} - \frac{1}{\sqrt{e}}\right)e^{-\frac{1}{2}\left(\frac{t-l}{s}\right)^2}$$





Lagrangian

$$-\frac{1}{2}am\dot{x}^{\mu}\dot{x}_{\mu}-aq\dot{x}^{\mu}A_{\mu}(x)-ab\frac{m}{2}$$

$$\frac{d}{d\tau}\frac{\delta L}{\delta\dot{x}^{\mu}}-\frac{\delta L}{\delta x^{\mu}}=0$$

$$\mathcal{L}\left(t,\vec{x},\dot{t},\dot{\vec{x}}\right)=-\frac{1}{2}m\dot{t}^{2}+\frac{1}{2}m\dot{\vec{x}}\cdot\dot{\vec{x}}-q\dot{t}\Phi\left(t,\vec{x}\right)+q\dot{x}_{j}A_{j}\left(t,\vec{x}\right)-\frac{1}{2}m$$

$$\vec{E}=-\nabla\Phi-\frac{\partial\vec{A}}{\partial t},\vec{B}=\nabla\times\vec{A}$$

$$m\ddot{t}=q\vec{E}\cdot\dot{\vec{x}}$$

$$m\ddot{\vec{x}}=q\dot{t}\vec{E}+q\dot{\vec{x}}\times\vec{B}$$

Convergence

$$K_{BA} = \lim_{N\to\infty} C_N \int \prod_{n=1}^{n=N} dt_n d\vec{x}_n e^{\imath\varepsilon \sum_{j=1}^{N+1} L_j}$$

$$\mathcal{L}_j \equiv \tilde{\mathcal{L}}_j^t + \overline{\mathcal{L}}_j^{\vec{x}} + \mathcal{L}_j^m$$

$$\tilde{\mathcal{L}}_j^t \equiv -a\frac{m}{2}\left(\frac{t_j - t_{j-1}}{\varepsilon}\right)^2 - qa\frac{t_j - t_{j-1}}{\varepsilon}\frac{\Phi\left(x_j\right)+\Phi\left(x_{j-1}\right)}{2}$$

$$\overline{\mathcal{L}}_j^{\vec{x}} \equiv a\frac{m}{2}\left(\frac{\vec{x}_j - \vec{x}_{j-1}}{\varepsilon}\right)^2 + qa\frac{\vec{x}_j - \vec{x}_{j-1}}{\varepsilon}\cdot\frac{\vec{A}\left(x_j\right)+\vec{A}\left(x_{j-1}\right)}{2}$$

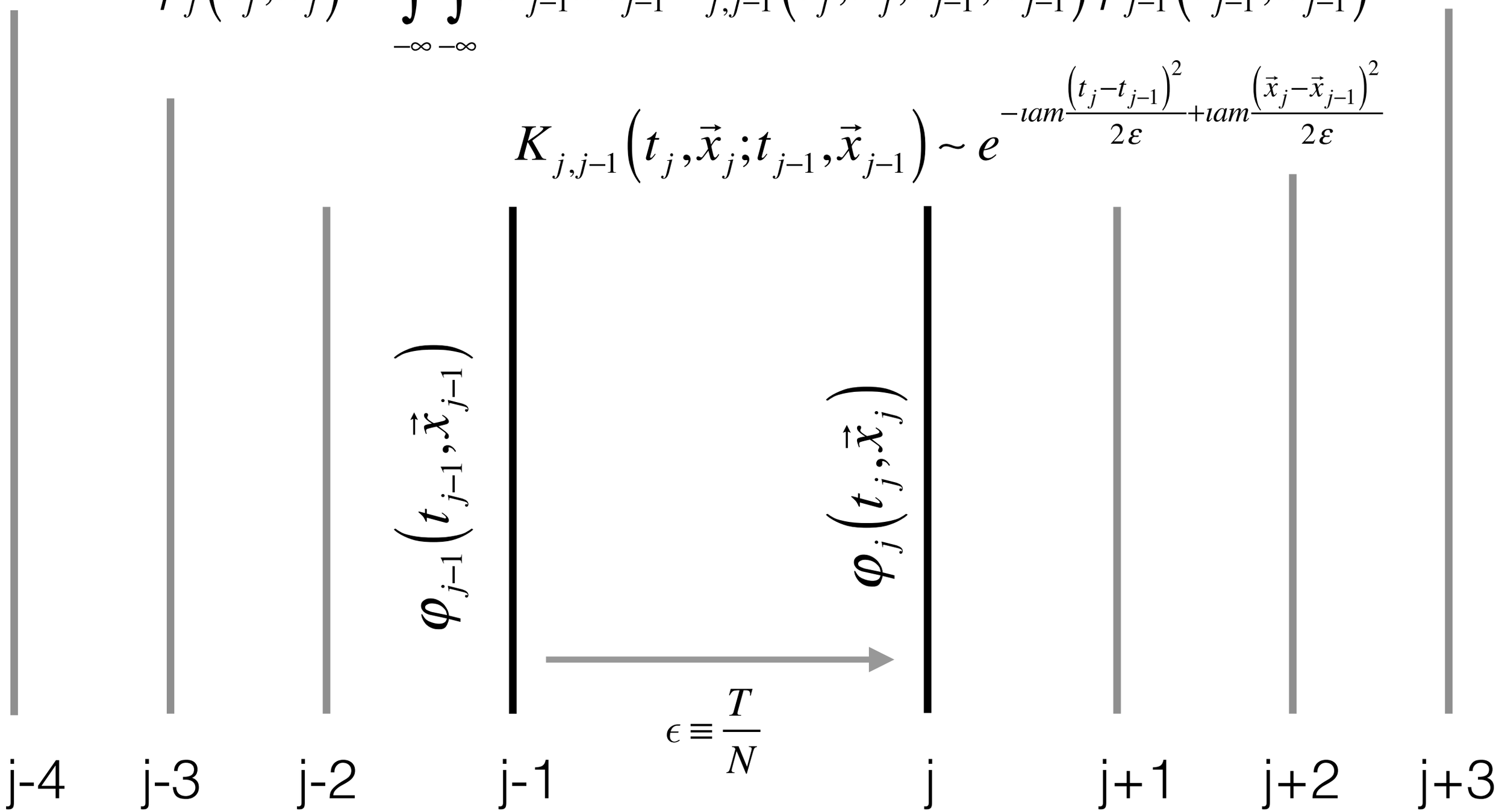
$$\mathcal{L}_j^m \equiv -ab\frac{m}{2}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dt_j d\vec{x} e^{-\imath am\frac{\left(t_j - t_{j-1}\right)^2}{2\varepsilon} + \imath am\frac{\left(\vec{x}_j - \vec{x}_{j-1}\right)^2}{2\varepsilon}}$$

Single slice

$$\varphi_j\left(t_j,\vec{x}_j\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dt_{j-1}d\vec{x}_{j-1}K_{j,j-1}\left(t_j,\vec{x}_j;t_{j-1},\vec{x}_{j-1}\right)\varphi_{j-1}\left(t_{j-1},\vec{x}_{j-1}\right)$$

$$K_{j,j-1}\left(t_j,\vec{x}_j;t_{j-1},\vec{x}_{j-1}\right)\sim e^{-\imath am\frac{\left(t_j-t_{j-1}\right)^2}{2\varepsilon}+\imath am\frac{\left(\vec{x}_j-\vec{x}_{j-1}\right)^2}{2\varepsilon}}$$



Normalization

$$\tilde{\varphi}_0(t_0) \equiv \sqrt[4]{\frac{1}{\pi\sigma_t^2}} e^{-\imath E_0 t_0 - \frac{(t_0-\bar{t}_0)^2}{2\sigma_t^2}}$$

$$\tilde{\varphi}_\varepsilon(t_1) = \int dt_0 e^{-\imath\frac{am}{2\varepsilon}(t_1-t_0)^2} \tilde{\varphi}_0(t_0)$$

$$\varphi_\varepsilon(t_1) = \sqrt{\frac{2\pi\varepsilon}{\imath am}} \sqrt[4]{\frac{1}{\pi\sigma_0^2}} \sqrt{\frac{1}{f_\varepsilon^{(t)}}} e^{-\imath E_0 t_1 + \imath\frac{E_0^2}{2am}\varepsilon - \frac{1}{2\sigma_0^2 f_\varepsilon^{(t)}}\left(t_1-\bar{t}_0-\frac{E_0}{am}\varepsilon\right)^2}$$

$$f_\tau^{(t)} \equiv 1 - \imath\frac{\tau}{m\sigma_t^2}$$

$$1 = \int dt_1 \tilde{\varphi}_\varepsilon^*(t_1)\tilde{\varphi}_\varepsilon(t_1)$$

$$C_N \equiv \sqrt{\frac{\imath am}{2\pi\varepsilon}}^{N+1}$$

Path integral kernel

$$K_\tau\left(x'';x'\right)=\lim_{N\to\infty}\int \mathcal{D}x e^{\iota\sum_{j=1}^{N+1}L_j\epsilon}$$

$$\mathcal{L}_j\equiv -am\frac{\left(x_j-x_{j-1}\right)^2}{2\varepsilon^2}-aq\frac{x_j-x_{j-1}}{\epsilon}\frac{A\left(x_j\right)+A\left(x_{j-1}\right)}{2}-b\frac{m}{2}$$

$$\mathcal{D}x\equiv\left(-\iota\frac{m^2}{4\pi^2a^2\varepsilon^2}\right)^{N+1}\prod_{n=1}^{n=N}d^4x_n$$

Schrödinger equation

$$\xi \equiv x_j - x_{j+1} \qquad\qquad \xi \sim \sqrt{\epsilon}$$

$$\psi_{\tau+\varepsilon}\left(x_{j+1}\right) = \sqrt{\frac{\imath am}{2\pi\varepsilon}}\sqrt{-\frac{\imath am}{2\pi\varepsilon}}^{3}\int d^4\xi e^{-\frac{\imath am\xi^2}{2\varepsilon}-\imath ab\frac{m}{2}\varepsilon}$$

$$\times e^{\imath aq\xi^\nu\left(A_\nu\left(x_{j+1}\right)+\frac{1}{2}\left(\xi^\mu\partial_\mu\right)A_\nu\left(x_{j+1}\right)+\ldots\right)}$$

$$\times\left(\psi_\tau\left(x_{j+1}\right)+\left(\xi^\mu\partial_\mu\right)\psi_\tau\left(x_{j+1}\right)+\frac{1}{2}\xi^\mu\xi^\nu\partial_\mu\partial_\nu\psi_\tau\left(x_{j+1}\right)+\ldots\right)$$

$$\imath\frac{d\psi_\tau}{d\tau}(t,\vec{x}) = -\frac{1}{2ma}\left(\left(\imath\partial_\mu - aqA_\mu(t,\vec{x})\right)\left(\imath\partial^\mu - aqA^\mu(t,\vec{x})\right) - a^2bm^2\right)\psi_\tau(t,\vec{x})$$

$$\imath\frac{d\psi_\tau}{d\tau} = -\frac{1}{2ma}\left(\left(\hat{p}_\mu - aqA_\mu\right)\left(\hat{p}^\mu - aqA^\mu\right) - a^2bm^2\right)\psi_\tau$$

Long, Slow Approximation

$$\imath \frac{d\psi_\tau(x)}{d\tau} \approx 0$$

$$\left(p^2 - a^2 b m^2\right)\psi \approx 0 \rightarrow a^2 b = 1$$

$$\left(\left(p - aqA\right)^2 - m^2\right) \approx 0 \rightarrow a = 1 \rightarrow b = 1$$

$$\imath \frac{d\psi_\tau}{d\tau} = -\frac{1}{2m}\left(\left(p_\mu - qA_\mu\right)\left(p^\mu - qA^\mu\right) - m^2\right)\psi_\tau$$

HOW LONG, HOW SLOW?

$$f \sim -\frac{E^2 - \vec{p}^2 - m^2}{2m}$$

$$E \sim m + \frac{\vec{p}^2}{2m}$$

$$E^2 - \vec{p}^2 - m^2 \sim \left(m + \frac{\vec{p}^2}{2m} \right)^2 - \vec{p}^2 - m^2 = \left(\frac{\vec{p}^2}{2m} \right)^2$$

$$\frac{\vec{p}^2}{2m} \sim eV$$

$$f \sim \frac{eV^2}{MeV} \sim eV \cdot 10^{-6}$$

PICOSECONDS

Rest Frame of the Vacuum

$$\left(G^{\mu\nu} + 8\pi G T^{\mu\nu}\right)_{;\nu} = 0$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

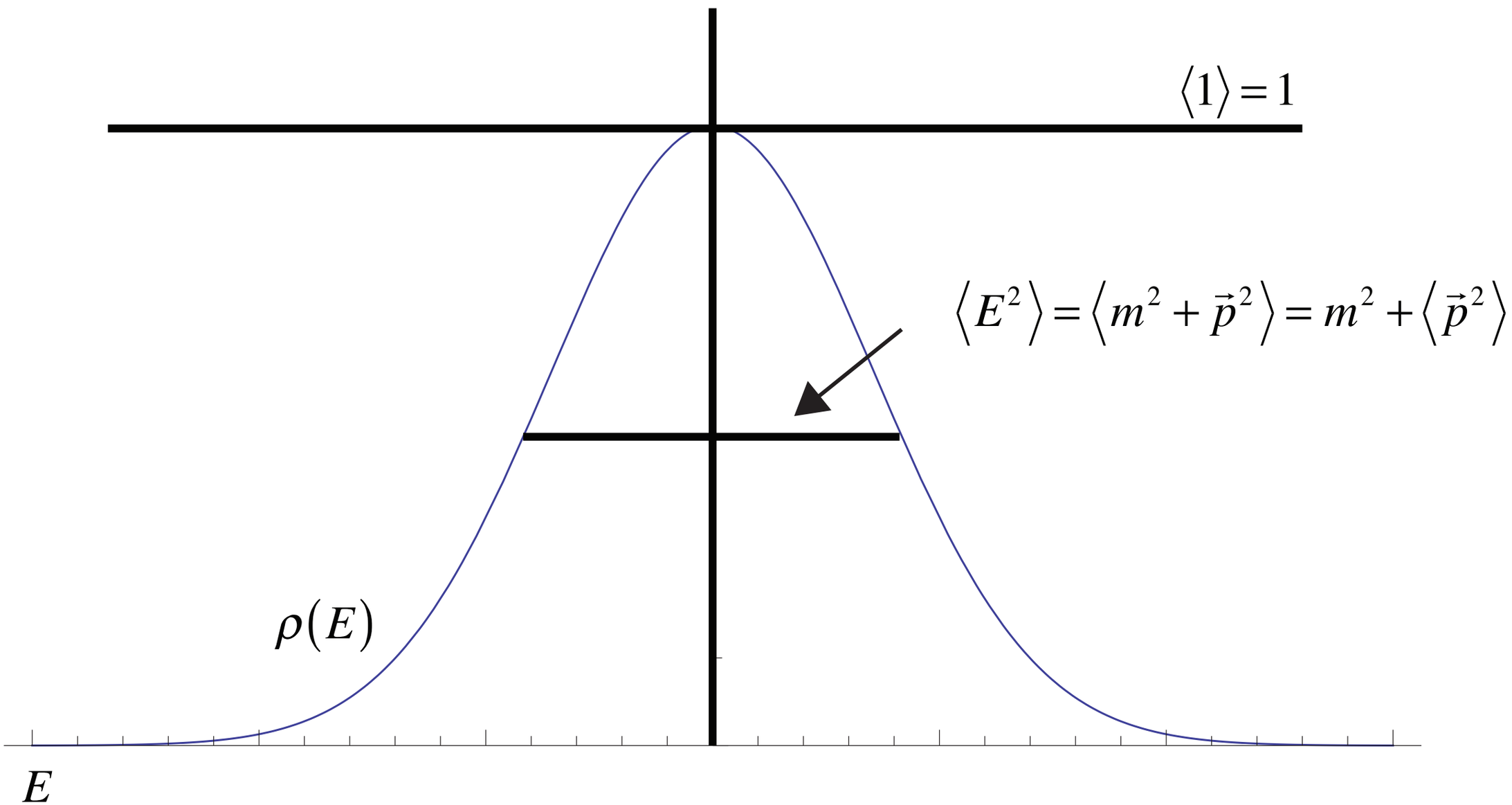
$$R^{(1)}_{\mu\nu} \equiv \frac{1}{2}\left(\frac{\partial^2 h^\lambda_\lambda}{\partial x^\mu \partial x^\nu} - \frac{\partial^2 h^\lambda_\mu}{\partial x^\lambda \partial x^\kappa} - \frac{\partial^2 h^\lambda_\kappa}{\partial x^\lambda \partial x^\mu} + \frac{\partial^2 h_{\mu\nu}}{\partial x^\lambda \partial x_\lambda}\right)$$

$$R^{(1)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\kappa} R^{(1)\lambda}_\lambda = -8\pi G\left(T_{\mu\nu} - t_{\mu\nu}\right)$$

$$t_{\mu\nu} \equiv \frac{1}{8\pi G}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\kappa} R^\lambda_\lambda - R^{(1)}_{\mu\nu} + \frac{1}{2}\eta_{\mu\kappa} R^{(1)\lambda}_\lambda\right)$$

Maximum entropy

$$\bar{E} \equiv \langle E \rangle = \sqrt{m^2 + \langle \vec{p} \rangle^2}$$



Free kernel

$$\left(\imath\frac{\partial}{\partial\tau}+\frac{E^2-p_x^2-m^2}{2m}\right)\hat{K}_\tau\left(p;p'\right)=\delta^{(4)}\left(p-p'\right)\delta\left(\tau\right)$$

$$\hat{K}_\tau\left(p;p'\right)=\exp\left(\imath\frac{p^2-m^2}{2m}\tau\right)\delta^{(4)}\left(p-p'\right)\theta\left(\tau\right)$$

$$K_\tau\left(x;x'\right)=-\imath\frac{m^2}{4\pi^2\tau^2}e^{-\frac{\imath m}{2\tau}\left(x-x'\right)^2-\imath\frac{m}{2}\tau}\theta\left(\tau\right)$$

$$\hat{\mathcal{K}}_s\left(p;p'\right)\equiv\int_0^\tau d\tau\exp\left(-s\tau\right)\hat{K}_\tau\left(p;p'\right)=\frac{2m\imath}{p^2-m^2+2m\imath s}$$

$$\hat{\bar{\mathcal{K}}}_s\left(p;p'\right)=\frac{\imath}{p^2-m^2+\imath\varepsilon}$$

$$\tau\to\infty\Rightarrow s\to 0$$

Evolution of the wave function

$$\imath\frac{\partial\psi_\tau}{\partial\tau}(t,x)=\left(\frac{\partial_t^2}{2m}-\frac{\partial_x^2}{2m}+\frac{m}{2}\right)\psi_\tau(t,x)\Rightarrow\psi_\tau(t,x)=\tilde{\psi}_\tau(t)\bar{\psi}_\tau(x)$$

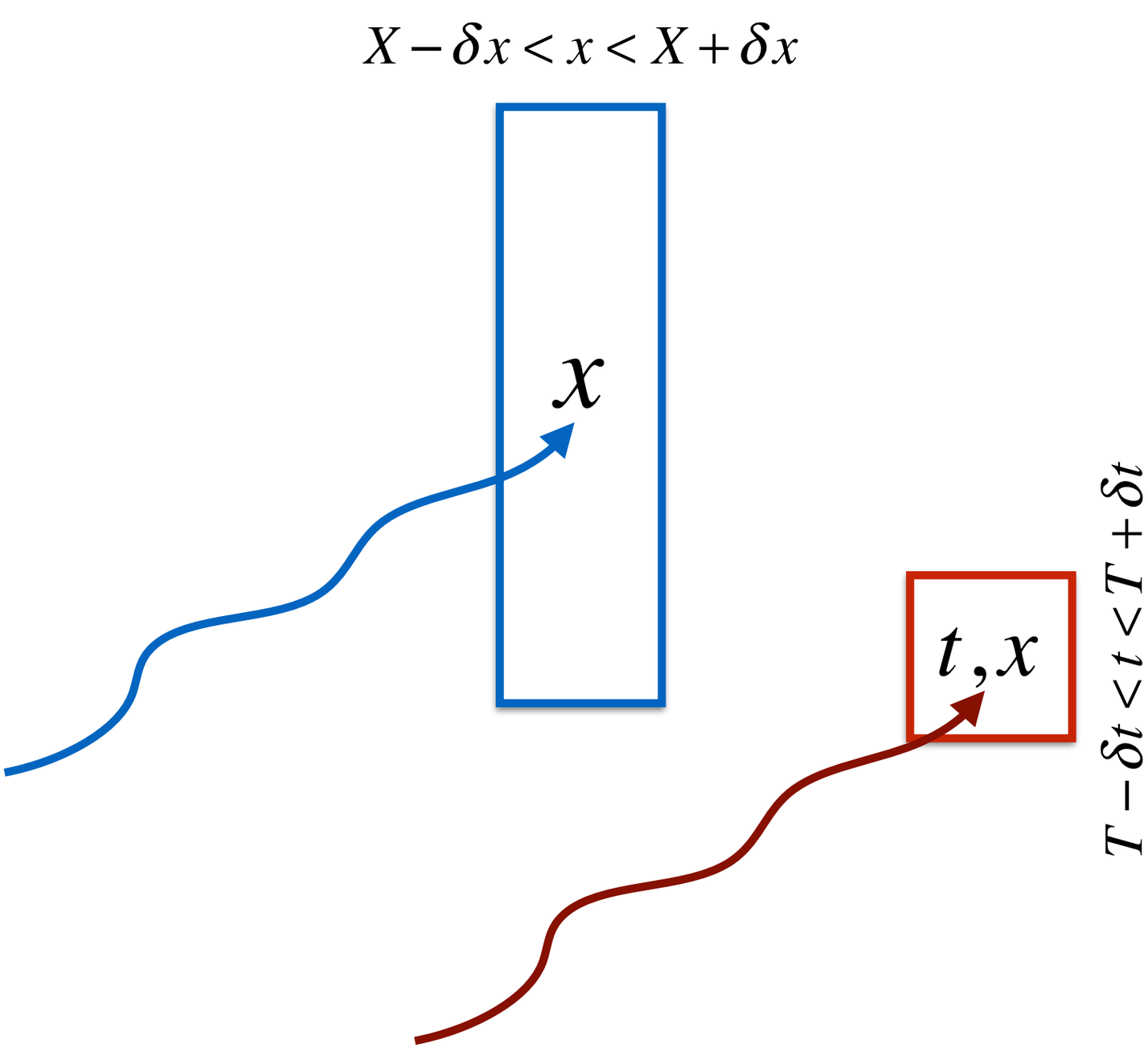
$$\bar{\varphi}_0(x)\equiv\sqrt[4]{\frac{1}{\pi\sigma_x^2}}e^{\imath p_0(x-x_0)-\frac{(x-x_0)^2}{2\sigma_x^2}}$$

$$\bar{\varphi}_\tau(x)=\sqrt[4]{\frac{1}{\pi\sigma_x^2}}\sqrt{\frac{1}{f_\tau^{(x)}}}e^{\imath p_0 x-\frac{1}{2\sigma_x^2 f_\tau^{(x)}}\left(x-x_0-\frac{p_0}{m}\tau\right)^2-\imath\frac{p_0^2}{2m}\tau},f_\tau^{(x)}\equiv1+\imath\frac{\tau}{m\sigma_x^2}$$

$$\tilde{\varphi}_0(t)\equiv\sqrt[4]{\frac{1}{\pi\sigma_t^2}}e^{-\imath E_0(t-t_0)-\frac{(t-t_0)^2}{2\sigma_t^2}}$$

$$\tilde{\varphi}_\tau(t)=\sqrt[4]{\frac{1}{\pi\sigma_t^2}}\sqrt{\frac{1}{f_\tau^{(t)}}}e^{-\imath E_0 t+\imath\frac{E_0^2}{2m}\tau-\frac{1}{2\sigma_t^2 f_\tau^{(t)}}\left(t-t_0-\frac{E_0}{m}\tau\right)^2},f_\tau^{(t)}\equiv1-\imath\frac{\tau}{m\sigma_t^2}$$

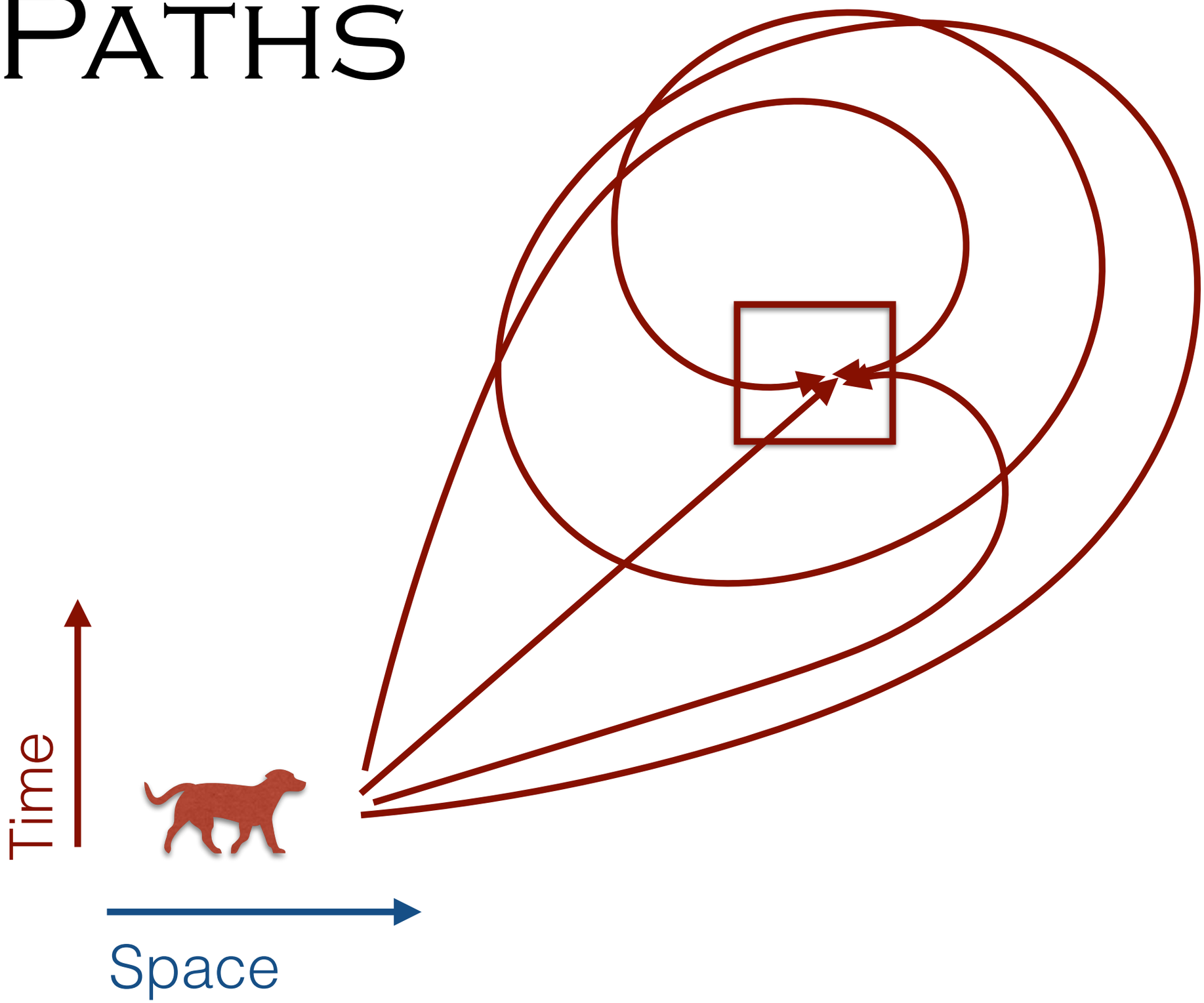
Definition of measurement



- Paths only known at ends
- At measurement in space, $x = X$.
- Therefore at measurement in time, $t = T$.

$$\langle t \rangle = \tau$$

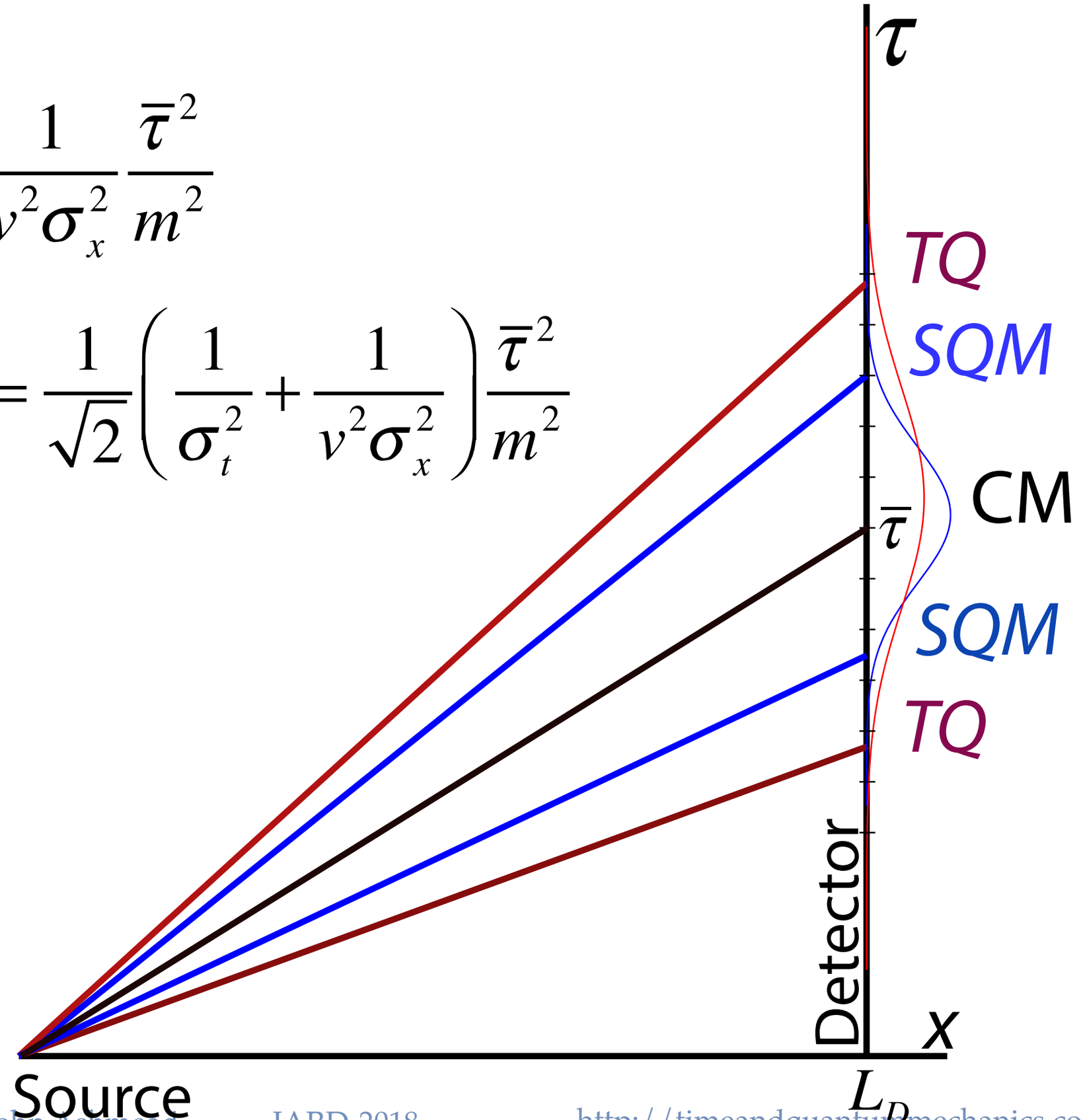
Simple and complex Paths



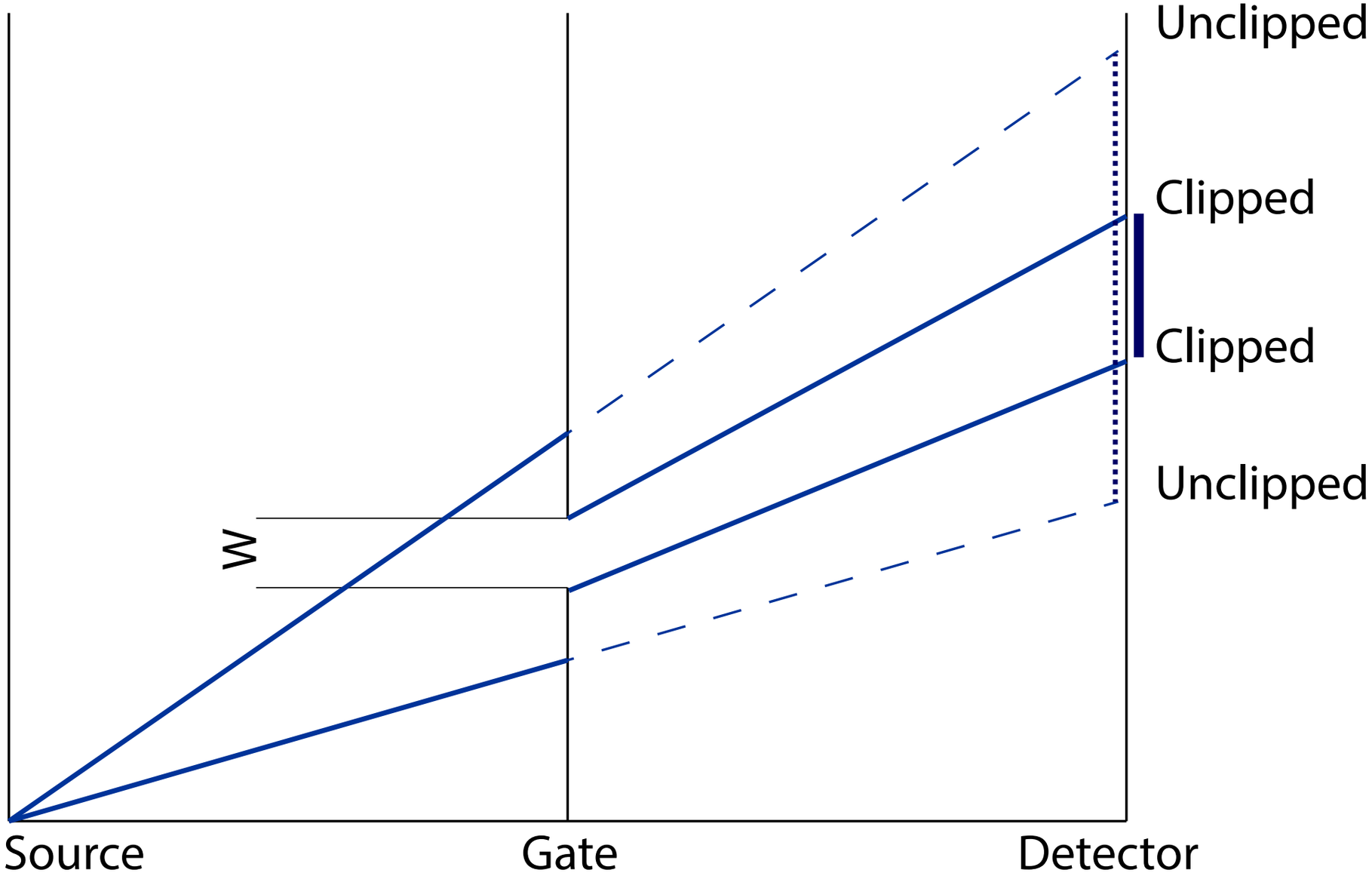
Time-of-Arrival

$$(\Delta t)^2 = \frac{1}{\sqrt{2}}\bar{\sigma}_\tau^2 = \frac{1}{\sqrt{2}}\frac{1}{v^2\sigma_x^2}\frac{\bar{\tau}^2}{m^2}$$

$$(\Delta t)^2 = \frac{1}{\sqrt{2}}\left(\tilde{\sigma}_\tau^2 + \bar{\sigma}_\tau^2\right) = \frac{1}{\sqrt{2}}\left(\frac{1}{\sigma_t^2} + \frac{1}{v^2\sigma_x^2}\right)\frac{\bar{\tau}^2}{m^2}$$

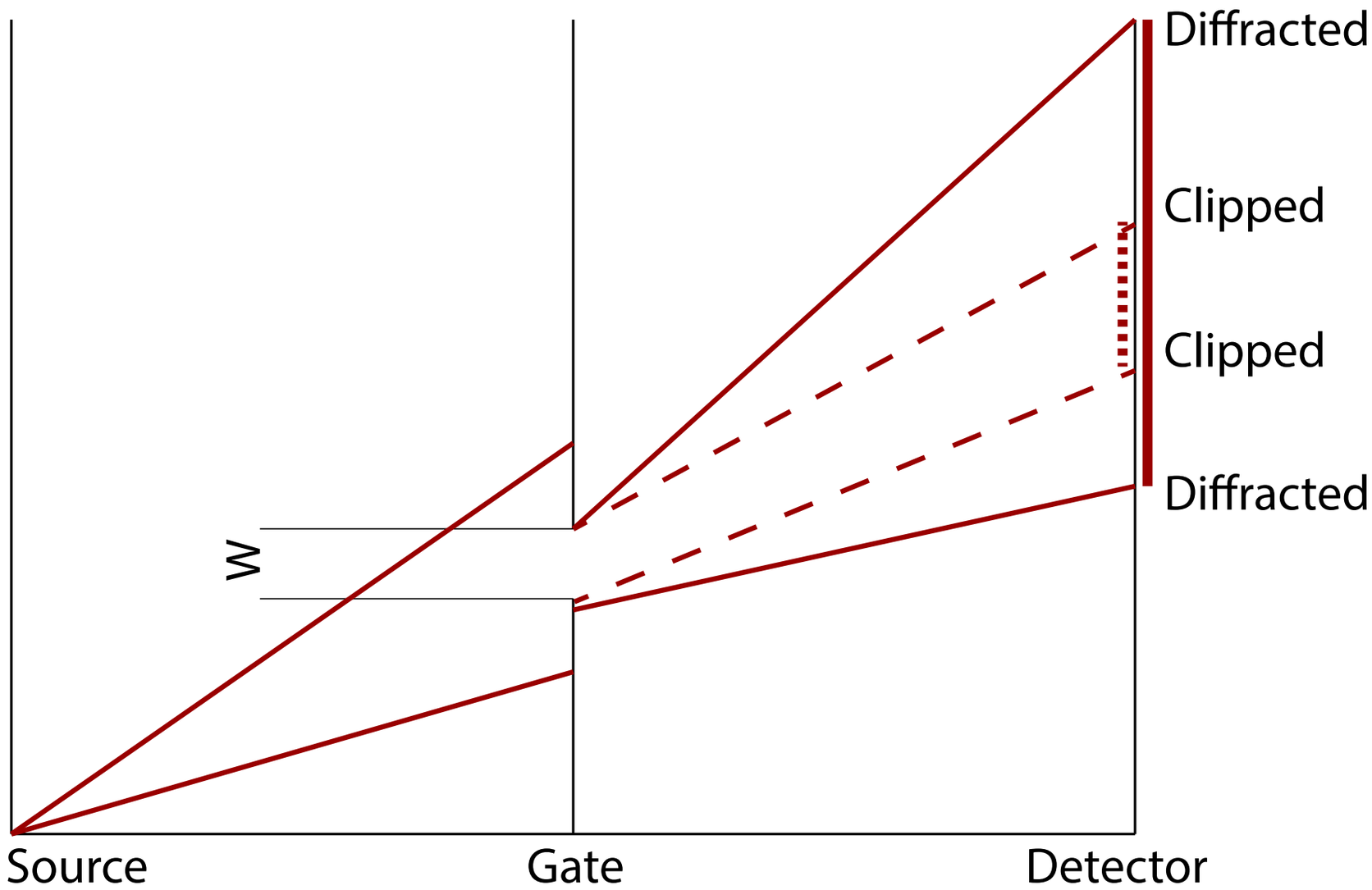


Single slit in SQM



- No uncertainty principle in time/energy
- Fast shutter constricts gate
- Arbitrarily small dispersion

Single slit in TQ

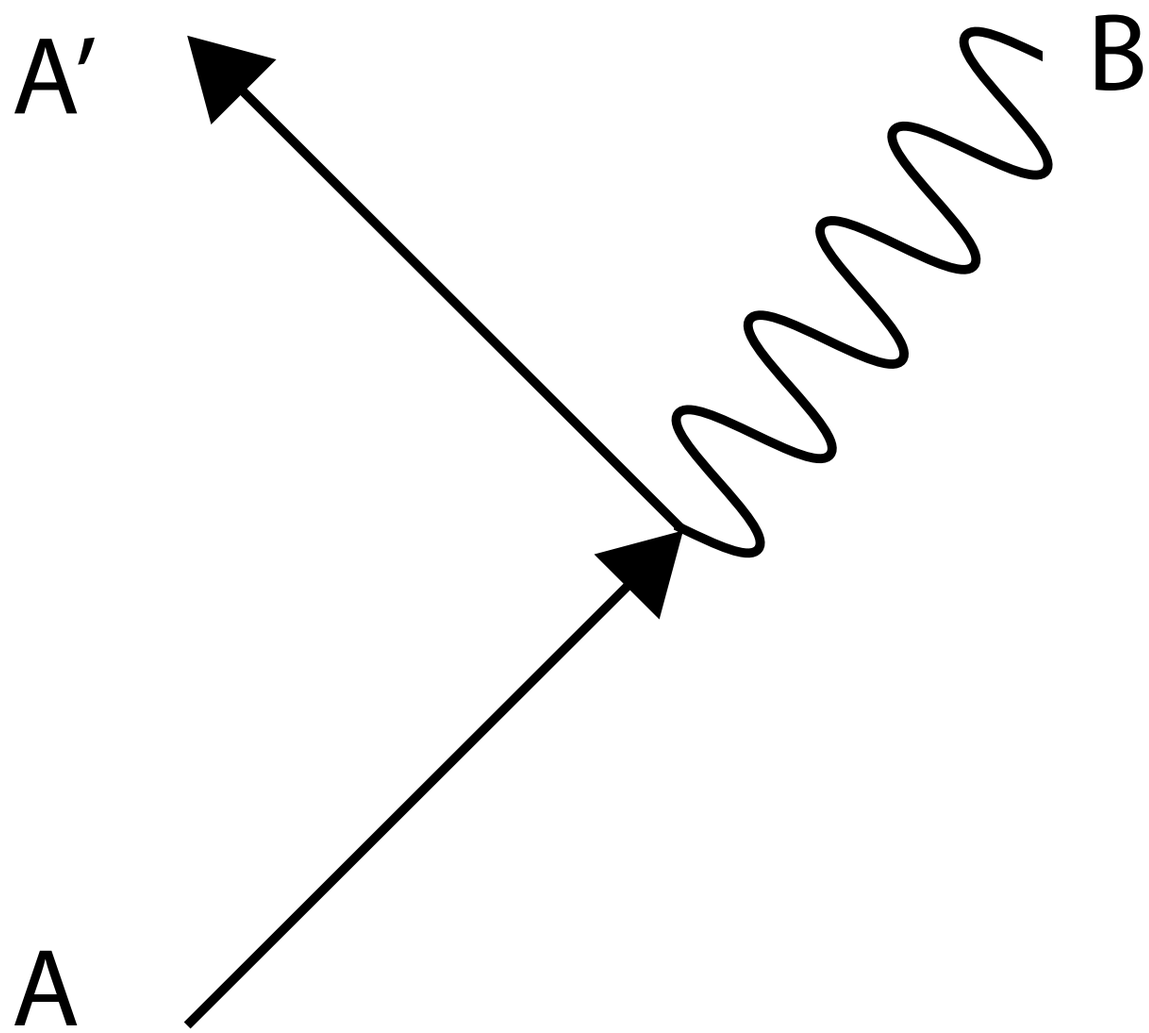


- Uncertainty principle in time/energy
- Diffraction
- Arbitrarily large dispersion

Why Field theory?

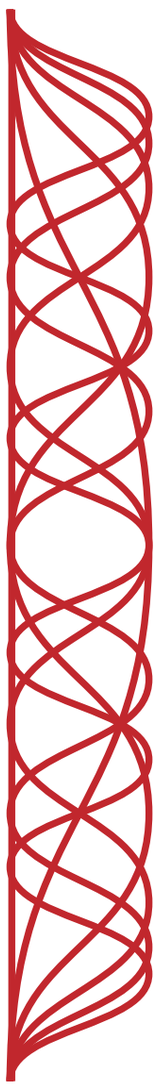
- Need short times, therefore high energies
- Need high velocities, therefore high energies.
- Need high energies, therefore will see particle creation.
- Particle creation implies multiple particles.
- Multiple particles implies field theory.
- Further, with one “extra” dimension, can TQ be renormalized?
- And there is the intrinsic interest of the question.

A/B MODEL



- A massive spinless particle (electron).
- B less massive spinless (photon).
- A emits and absorbs B′s.

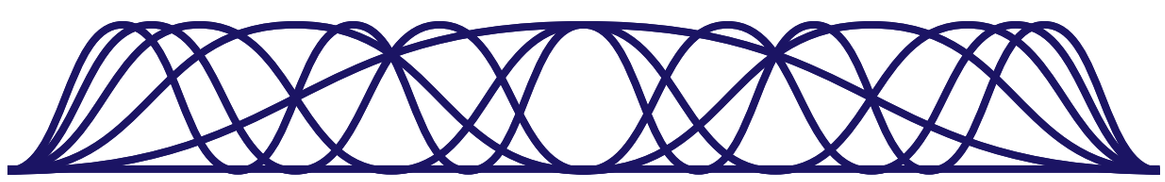
Fock Space



$$a^{\dagger}_{\vec{k}}\left|n_{\vec{k}}\right\rangle = \sqrt{n_{\vec{k}}+1}\left|n_{\vec{k}}+1\right\rangle$$

$$a_{\vec{k}}\left|n_{\vec{k}}\right\rangle = \sqrt{n_{\vec{k}}}\left|n_{\vec{k}}-1\right\rangle$$

$$\left[a_{\vec{k}}, a^{\dagger}_{\vec{k}'}\right] = \delta_{\vec{k}\vec{k}'}$$



- Box from x=-L to x=+L
- 4D Box from t=-L to t=+L
- Clock time from 0 to T. Need L>>T.

Anti-symmetry in Time

$$\phi_{kk'}(1,2) \equiv \frac{1}{\sqrt{2}}\left(\phi_k(1)\phi_{k'}(2) + \phi_k(2)\phi_{k'}(1)\right)$$

$$\varphi_{sym}(1,2) = \tilde{\varphi}_{sym}(1,2)\bar{\varphi}_{sym}(1,2) + \tilde{\varphi}_{anti}(1,2)\bar{\varphi}_{anti}(1,2)$$

Lagrangian

$$\overline{\mathcal{L}}\left[\phi,\phi^*\right]=\frac{1}{2}\frac{\partial\phi^*}{\partial\tau}\frac{\partial\phi}{\partial\tau}-\frac{1}{2}\nabla\phi^*\nabla\phi-\frac{m^2}{2}\phi^*\phi$$

$$L_0=\frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi-\frac{m^2}{2}\phi^*\phi=\frac{1}{2}\partial_t\phi^*\partial_t\phi-\frac{1}{2}\nabla\phi^*\nabla\phi-\frac{m^2}{2}\phi^*\phi$$

$$2m\imath\frac{d}{d\tau}\phi=-\left(\omega^2-\vec{k}^2-m^2\right)\phi$$

$$\mathcal{L}_{clock}=m\imath\left(\dot{\phi}^*\phi-\phi^*\dot{\phi}\right)$$

$$\mathcal{L}\left[\phi,\phi^*\right]=m\imath\left(\dot{\phi}^*\phi-\phi^*\dot{\phi}\right)+\frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi-\frac{m^2}{2}\phi^*\phi-V\left(\phi^*,\phi\right)$$

Generating function

$$K_T \equiv \int \mathcal{D}\phi \exp\left(\imath \int_0^T d\tau \int d^4x \mathcal{L}\left[\phi,\phi^*,\dot{\phi},\dot{\phi}^*\right]\right) \qquad \mathcal{D}\phi \equiv \prod_{n=1}^{N}\prod_x d\phi_n^*(x)\, d\phi_n(x)$$

$$\langle \psi' | \int \mathcal{D}\phi \exp\left(\imath \int_0^T d\tau \int d^4x \mathcal{L}\left[\phi,\phi^*,\dot{\phi},\dot{\phi}^*\right]\right) | \psi \rangle$$

$$Z[J] \equiv \langle 0 | \int \mathcal{D}\phi \exp\left(\imath \int_0^T d\tau \mathcal{L}\left[\phi,\phi^*,\dot{\phi},\dot{\phi}^*\right] + J_\tau[x]\phi(x) + J_\tau^*[x]\phi^*(x)\right) | 0 \rangle$$

$$Z[J] = Z[0]\exp\left(\imath W[J]\right)$$

$$K_{\tau'\tau}^{(2)}(x';x) = \left.\frac{\delta^2 W[J]}{\delta J_{\tau'}(x')\delta J_\tau(x)}\right|_{J_{\tau'},J_\tau=0}$$

Propagator

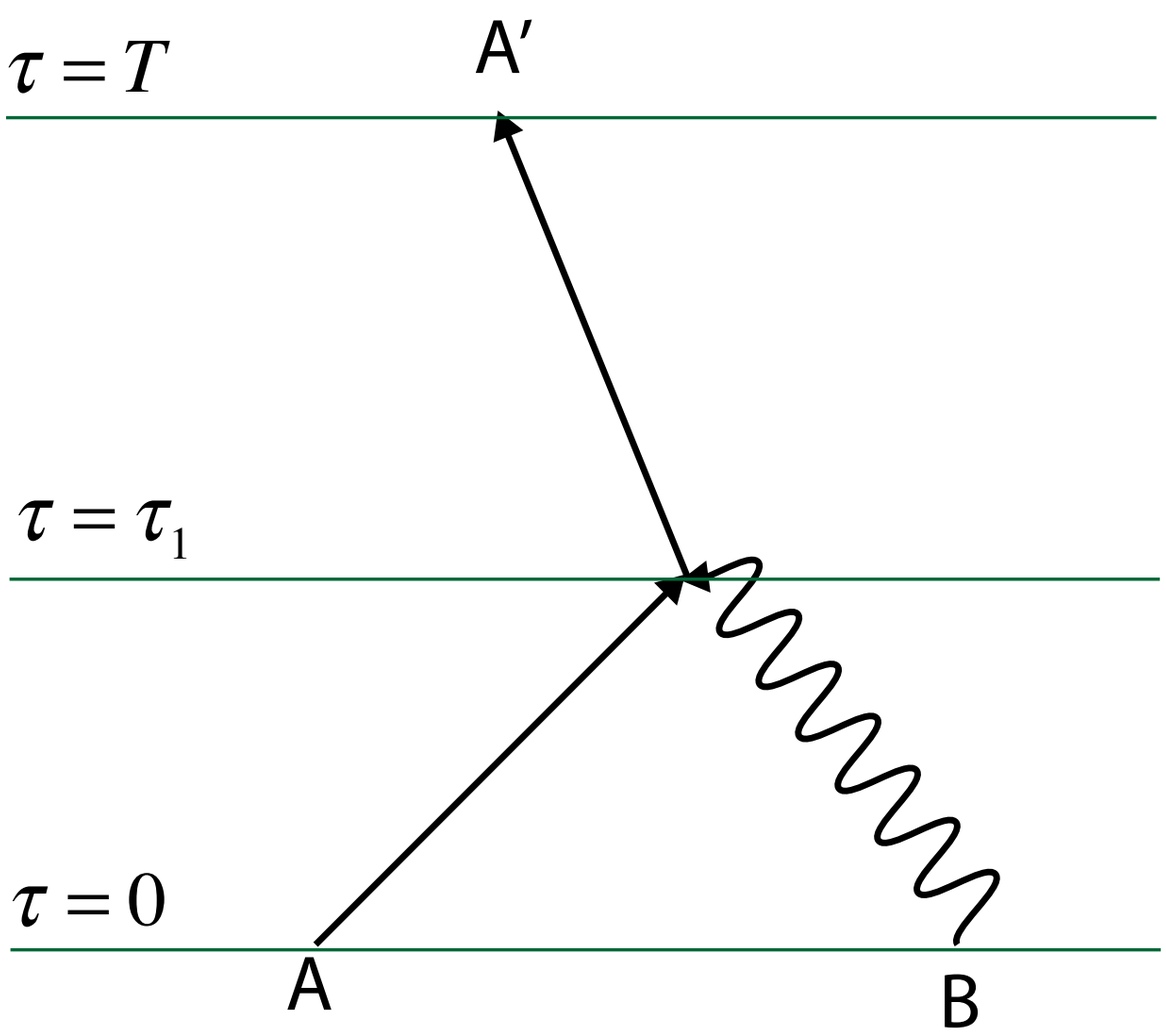
$$W[J] = -\frac{1}{2}\int d\tau\, d\tau' \int d^4x d^4x' J_\tau(x) D(x-x') J_{\tau'}(x')$$

$$\left(2m\imath\frac{\partial}{\partial\tau} - \left(\partial^\mu\partial_\mu + m^2\right)\right) D_{\tau\tau'}(x-x') = \delta(\tau-\tau')\delta^{(4)}(x-x')$$

$$D_\tau(p,p') = \frac{1}{2m}\exp\left(\imath\frac{p^2-m^2}{2m}\tau\right)\delta^4(p-p')\theta(\tau)$$

$$D_s(p,p') = \int_0^\infty d\tau \exp(-s\tau) D_\tau(p,p') = \frac{\imath}{p^2-m^2+2m\imath s}$$

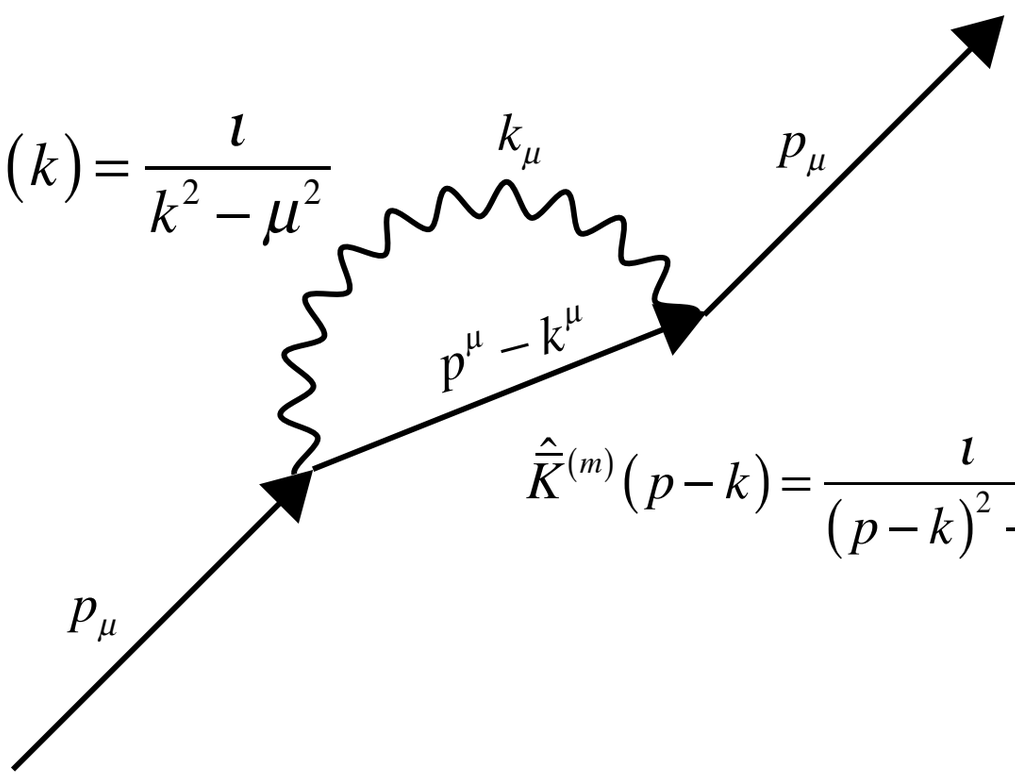
Feynman Rules



- Topology unchanged
- Every SQM propagator replaced by a TQ propagator (fuzzier)
- The four dimension energy-momentum delta function at each vertex is to be understood as in terms of the coordinate energy.
- Amplitudes are to be integrated over intermediate values of the clock time
- Only problems with finite time are interesting: no beams, cross-sections, rates, or averages.

Loop Correction: SQM

$$\hat{\bar{K}}^{(\mu)}(k) = \frac{\imath}{k^2 - \mu^2}$$

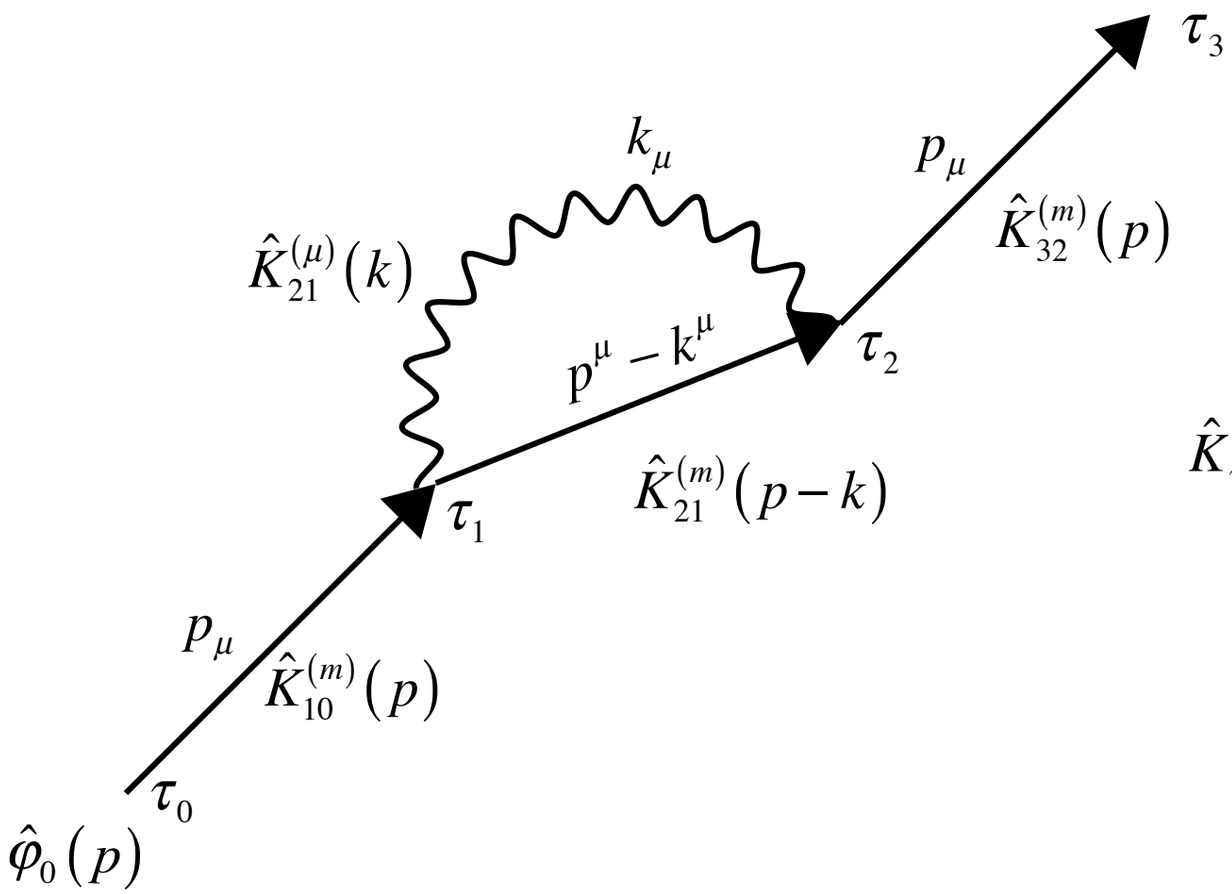


$$\hat{\bar{K}}^{(m)}(p-k) = \frac{\imath}{(p-k)^2 - m^2}$$

$$I \equiv \int d^4k \hat{\bar{K}}^{(m)}(p-k)\hat{\bar{K}}^{(\mu)}(k)$$

$$\int d^4k \frac{\imath}{(p-k)^2 - m^2}\frac{\imath}{k^2 - m^2} \sim \ln(\Lambda)$$

Loop Correction: TQ I



$$\hat{I}_\tau = \int d^4k \hat{K}_\tau^{(\mu)}(k)\hat{K}_\tau^{(m)}(p-k)$$

$$\hat{K}_\tau^{(\mu)}(p-k) = \frac{1}{2m}\exp\left(\imath\frac{(E-w)^2-\left(\vec{p}-\vec{k}\right)^2-m^2}{2m}\tau\right)$$

$$\hat{K}_\tau^{(\mu)}(k) = \frac{1}{2\mu}\exp\left(\imath\frac{w^2-\vec{k}^2-\mu^2}{2\mu}\tau\right)$$

$$\mathcal{I}_s(p) = \int d^4k \frac{i}{2\left(k^2(m+\mu)-2k\mu p+\mu\left(p^2-m(m+\mu-2is)\right)\right)}$$

Loop correction: TQ II

$$\tilde{\varphi}_2(t'') = \int dt'\, dt \tilde{K}_{21}(t'';t')\tilde{K}_{10}(t';t)\tilde{\varphi}_0(t)$$

$$\hat{I}_\tau = \frac{1}{4\mu m}\hat{I}_t\hat{I}_x\hat{I}_y\hat{I}_z \exp\left(-\imath\frac{m+\mu}{2}\tau\right)$$

$$\hat{I}_t \equiv \int dw\, dE \exp\left(\imath\frac{w^2}{2\mu}\tau\right)\exp\left(\imath\frac{(E-w)^2}{2m}\tau\right)\sqrt{\frac{1}{\pi\sigma_E^2}}\exp\left(\imath E t_0 - \frac{(E-E_0)^2}{2\sigma_E^2}\right)$$

$$E_0 \to t_0, E \to t, \sigma_E^2 \to \sigma_t^2, t_0 \to -E_0,$$
$$\tau \to M, m \to \tau_{10}, w \to t', \mu \to \tau_{21}$$

$$\hat{I}_t = \sqrt{\frac{2\pi\mu}{\imath\tau}}\sqrt{\frac{2\pi m}{\imath\tau}}\exp(\imath t_0 E_0)\sqrt[4]{\frac{1}{\pi\sigma_E^2}}\sqrt{\frac{1}{f_{m+\mu}^{(t)}}}e^{-\frac{E_0^2}{2\sigma_E^2 f_{m+\mu}^{(t)}}+\imath\frac{t_0^2}{2\tau f_{m+\mu}^{(t)}}(m+\mu)}$$

$$f_{m+\mu}^{(t)} = 1 - \imath\frac{m+\mu}{\tau\sigma_E^2}$$

Loop corrections III

$$\mathcal{I}_s(x';x) = \int_0^\infty d\tau \exp(-s\tau) K_\tau^{(m)}(x';x) K_\tau^{(\mu)}(x';x)$$

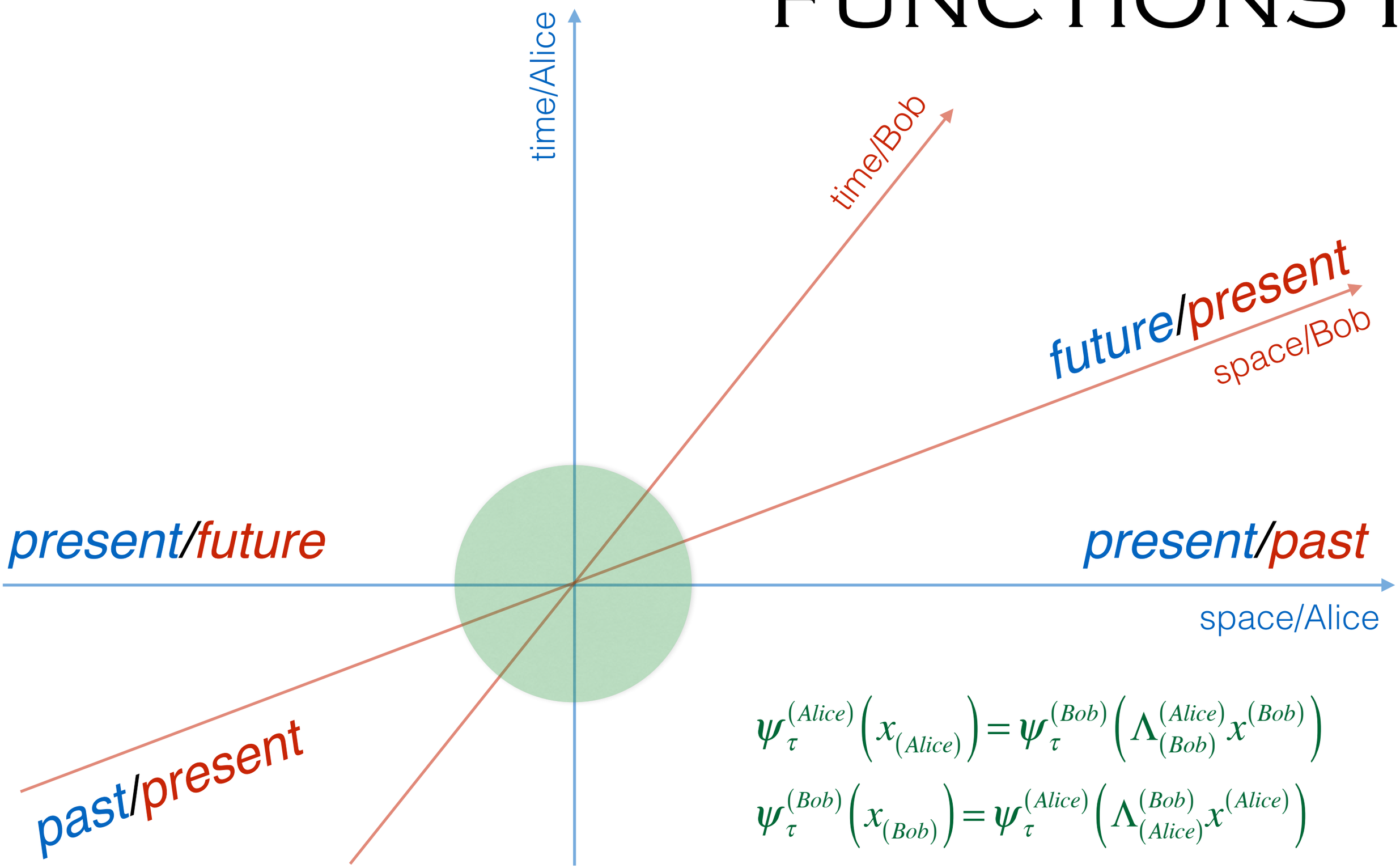
$$K_\tau^{(m)}(x';x) = -\frac{m^2}{4\pi^2\tau^2}\exp\left(-\frac{\imath m}{2\tau}(x''-x')^2 - \imath\frac{m}{2}\tau\right) \qquad \& m \to \mu$$

$$\mathcal{I}_s(x';x) = \frac{m^2}{4\pi^2}\frac{\mu^2}{4\pi^2}\int_0^\infty d\tau \frac{1}{\tau^4}\exp(-s\tau)\exp\left(-\imath\frac{m+\mu}{2}\tau - \imath\frac{m+\mu}{2\tau}(x'-x)^2\right)$$

$$\mathcal{I}_s(x';x) = c\int_0^\infty d\tau \frac{1}{\tau^4}\exp(-s\tau)\exp\left(-\imath a\tau - \imath\frac{b}{\tau}\right)$$

$$\mathcal{I}_s(x';x) = -\frac{m^2}{4\pi^2}\frac{\mu^2}{4\pi^2}\frac{2(\frac{m+\mu}{2}-\imath s)^2 K_3\left(2\sqrt{-\imath\frac{m+\mu}{2}(x''-x')^2}\sqrt{\imath\frac{m+\mu}{2}+s}\right)}{(-\imath\frac{m+\mu}{2}(x''-x')^2)^{3/2}\sqrt{\imath\frac{m+\mu}{2}+s}}$$

Alice & Bob Wave functions II



$$\psi_{\tau}^{(Alice)}\left(x_{(Alice)}\right)=\psi_{\tau}^{(Bob)}\left(\Lambda_{(Bob)}^{(Alice)}x^{(Bob)}\right)$$

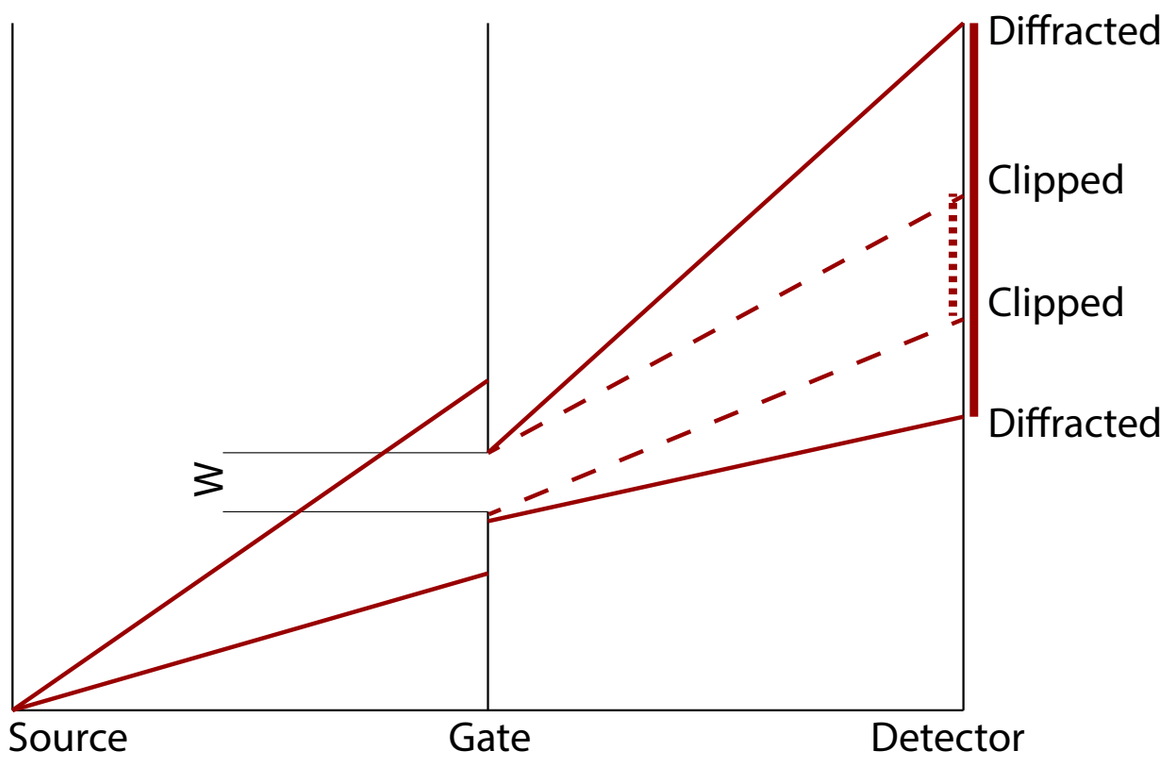
$$\psi_{\tau}^{(Bob)}\left(x_{(Bob)}\right)=\psi_{\tau}^{(Alice)}\left(\Lambda_{(Alice)}^{(Bob)}x^{(Alice)}\right)$$

Falsifiability

"You can have as much junk in the guess as you like, provided that the consequences can be compared with experiment."
— R. P. Feynman

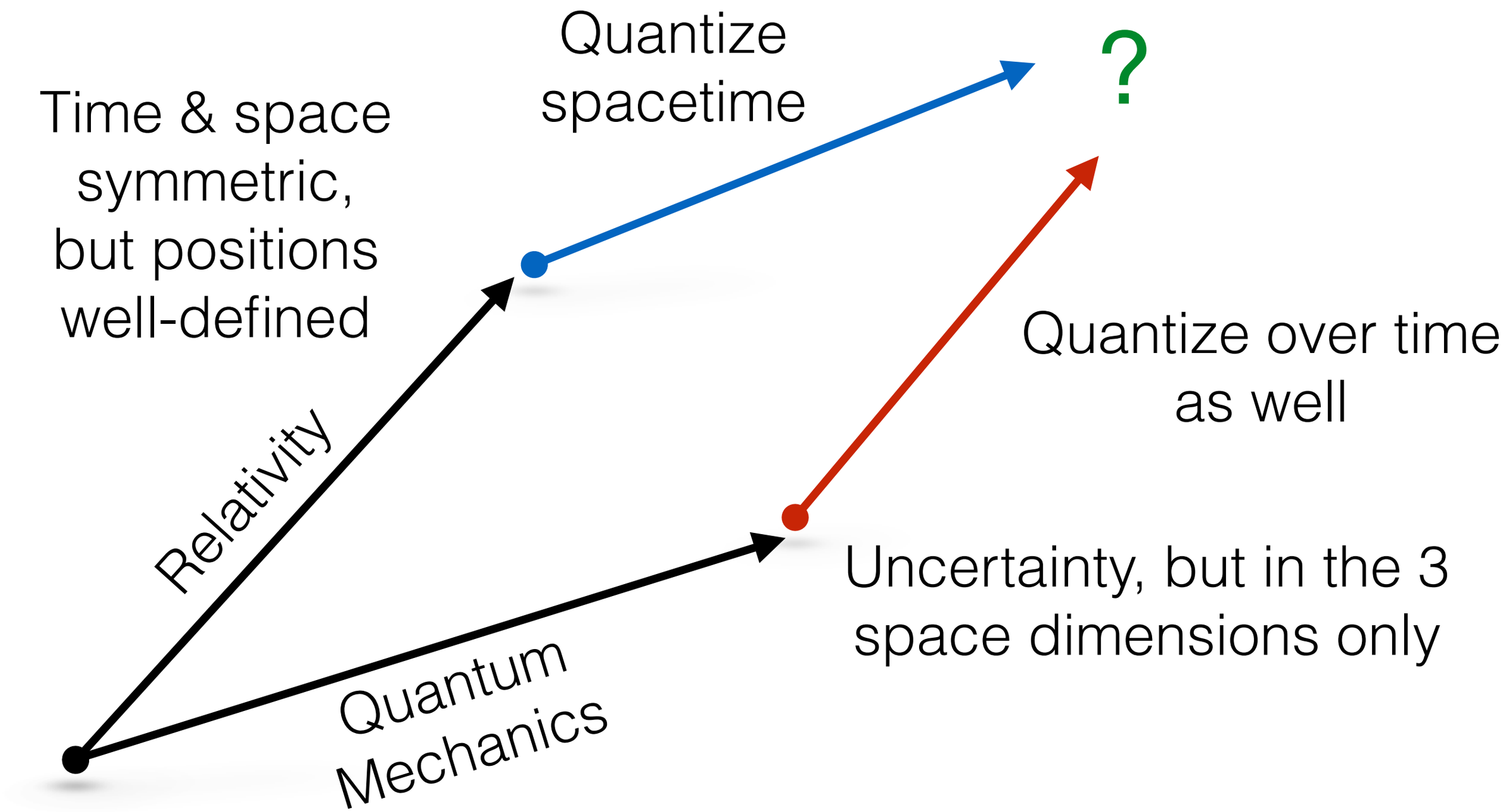
- Initial dispersion fixed by symmetry & maximum entropy
- Evolution fixed by long, slow approximation
- Choice of frame has only second order effects

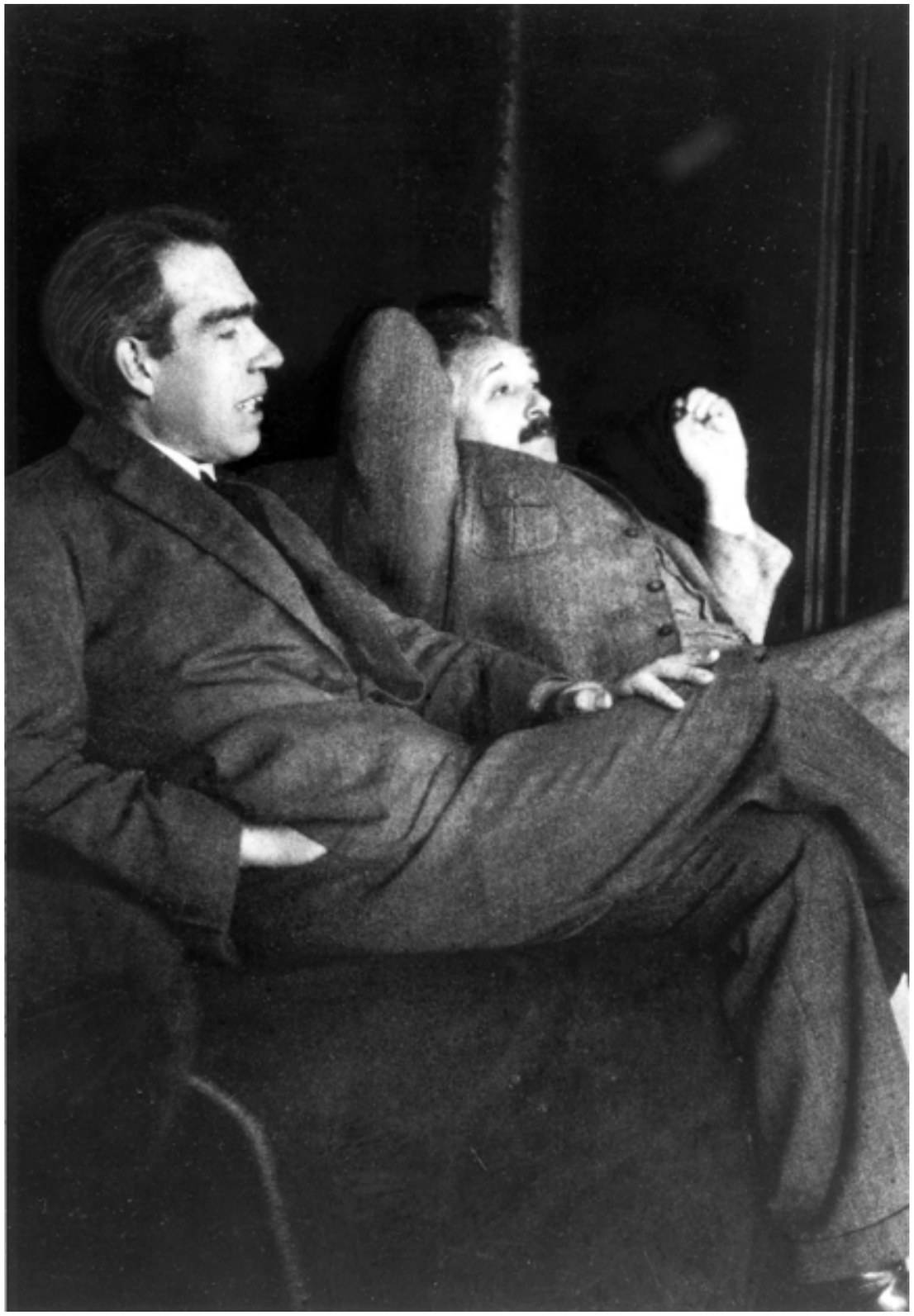
QM Effects in Time



- Dispersion in time
- Uncertainty in time/energy
- Wave functions anti-symmetric in time
- Cutoff dependence
- Shadowing in time
- Energy & time first class operators
- …

Mind the Gap


Time & space symmetric, but positions well-defined
Quantize spacetime
?
Quantize over time as well
Uncertainty, but in the 3 space dimensions only
Relativity
Quantum Mechanics



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