Time and Quantum Mechanics

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In quantum mechanics, time and space are handled very differently. For instance, assume we have a reference clock, a metronome, whose ticks establish a time τ. At each tick of the clock we have a wave function $\psi_\tau(\vec{x})$. We have implicitly assumed that we have an exact knowledge of the position of the particle in time (our clock ticks may be as frequent as we like) but that its position in space is uncertain. It is difficult to reconcile this with relativity, which would require a near complete symmetry between time and space: if one is uncertain, then so must be the other.

The resolution we look at here is to posit that the wave function should be extended to include time. At each clock tick τ, we hypothesize that the wave function should be a function of time as well as of space: $\psi_\tau(\vec{x}) \to \psi_\tau(t,\vec{x})$. We define the properties of this extended wave function by covariance. We show we do not need any additional assumptions to fully define the associated dynamics, to compute $\psi_{\tau+\Delta\tau}(t,\vec{x})$ given $\psi_\tau(t,\vec{x})$.

To do this we use path integrals. Normally the paths in path integrals are given as trajectories in space: $\vec{x}_\tau$; we extend the paths to include variation in time: $t_\tau,\vec{x}_\tau$. The resulting paths extend a bit into the future and into the past, as if time is fuzzy. We can think of this as the way a dog's path is usually ahead or behind its master's, but the average path is that of the master.

The sums in path integrals are weighed by the action for each path, by the integral of the Lagrangian over time: $\psi_{\tau'}(t_{\tau'},\vec{x}_{\tau'}) = \int \mathcal{D}t_\tau \mathcal{D}\vec{x}_\tau \exp\left(i\int_0^{\tau'} d\tau \mathcal{L}\left[t_\tau,\vec{x}_\tau\right]\right)\psi_0(t_0,\vec{x}_0)$.

Interestingly enough, we can reuse a Lagrangian $\mathcal{L}$ used in the classical mechanics treatment of a single relativistic particle. To ensure convergence of the path integrals we use Morlet wavelet analysis rather than Fourier analysis. As Morlet wavelets are built from Gaussians this lets us – without loss of generality – analyze an arbitrary wave function as a sum over Gaussians.

Usually path integrals are derived from the associated Schrödinger equation, but here it is more natural to derive the associated Schrödinger equation from the limit of the path integral as the duration of the clock tick goes to zero:

$$i\frac{\partial}{\partial\tau}\psi_\tau(t,\vec{x}) \equiv \lim_{\Delta\tau\to 0}\frac{\psi_{\tau+\Delta\tau}(t,\vec{x})-\psi_\tau(t,\vec{x})}{\Delta\tau}.$$

We get:

$$i\frac{\partial}{\partial\tau}\psi_\tau(t,\vec{x}) = -\frac{1}{2m}\left((p-eA)_\mu(p-eA)^\mu - m^2\right)\psi_\tau(t,\vec{x})$$

where $p_\mu = \left(i\frac{\partial}{\partial t}, i\vec{\nabla} \right)$ and the electromagnetic potential is $A_\mu = \left(\Phi, -\vec{A}\right)$. The right side will be recognized as the Klein-Gordon equation with the minimal substitution. Therefore quantum time reduces to the Klein-Gordon equation in the limit as the variation of the wave function with the clock time goes to zero, when $i\frac{\partial}{\partial \tau}\psi_\tau \approx 0$. We can understand this as meaning that over longer periods of time, for relatively slowly varying potentials, the detailed variations in the wave function average out. We call this the long, slow approximation and use it to define the transition from temporal quantization (the hypothesis that wave functions have extensions in time) to standard quantum theory. With respect to time, temporal quantization is to standard quantum theory as standard quantum theory is to classical mechanics.

We extend this approach to the multiple particle case (quantum electrodynamics) and then to the case of attractive potentials.

In the case of multiple particles, we get the usual results for the Feynman diagrams, with additional fuzziness in time. The loop calculations are particularly striking: in temporal quantization, we have an initial uncertainty in time Δt associated with any wave function. If we expand the loop results in powers of $\frac{1}{\Delta t}$ we get the usual ultraviolet divergences in the limit as $\Delta t \rightarrow 0$. By the Heisenberg uncertainty principle, $\Delta t \rightarrow 0 \Rightarrow \Delta E \rightarrow \infty$, so the source of the familiar ultraviolet divergences is precisely the assumption that time is flat, that $\Delta t = 0$. Similar results follow for the infrared divergences; they reappear when we take the limit as $\Delta t \rightarrow \infty$, also an unphysical assumption.

In the case of attractive potentials, the long, slow condition picks out the usual atomic orbitals. By dimensional arguments, the width in time of the resulting orbitals is of order the time taken by light to cross the atom, therefore of order attoseconds or less.

As current experimental time resolutions are just getting down to the attosecond level, this both explains why we might not have already seen fuzziness in time and offers the possibility of detecting it.

The greater simplicity of temporal quantization – manifest covariance & absence of singularities – suggests that it may be correct. Further, as temporal quantization is based on a strict application of covariance, experiments that refute it are likely to show results that are interesting in their own right: failures of Lorentz covariance, a local preferred frame for quantum mechanics, and the like. With respect to the experimental investigation of temporal quantization, we are therefore in the position of a bookie who has so arranged the odds so that no matter which horse wins, he will still come out ahead.